Basics of CT Image Reconstruction

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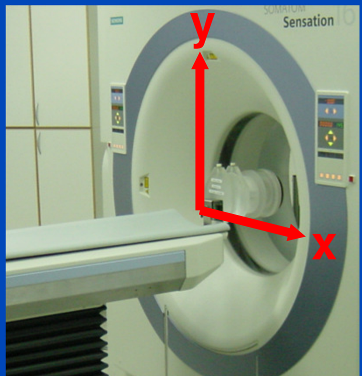
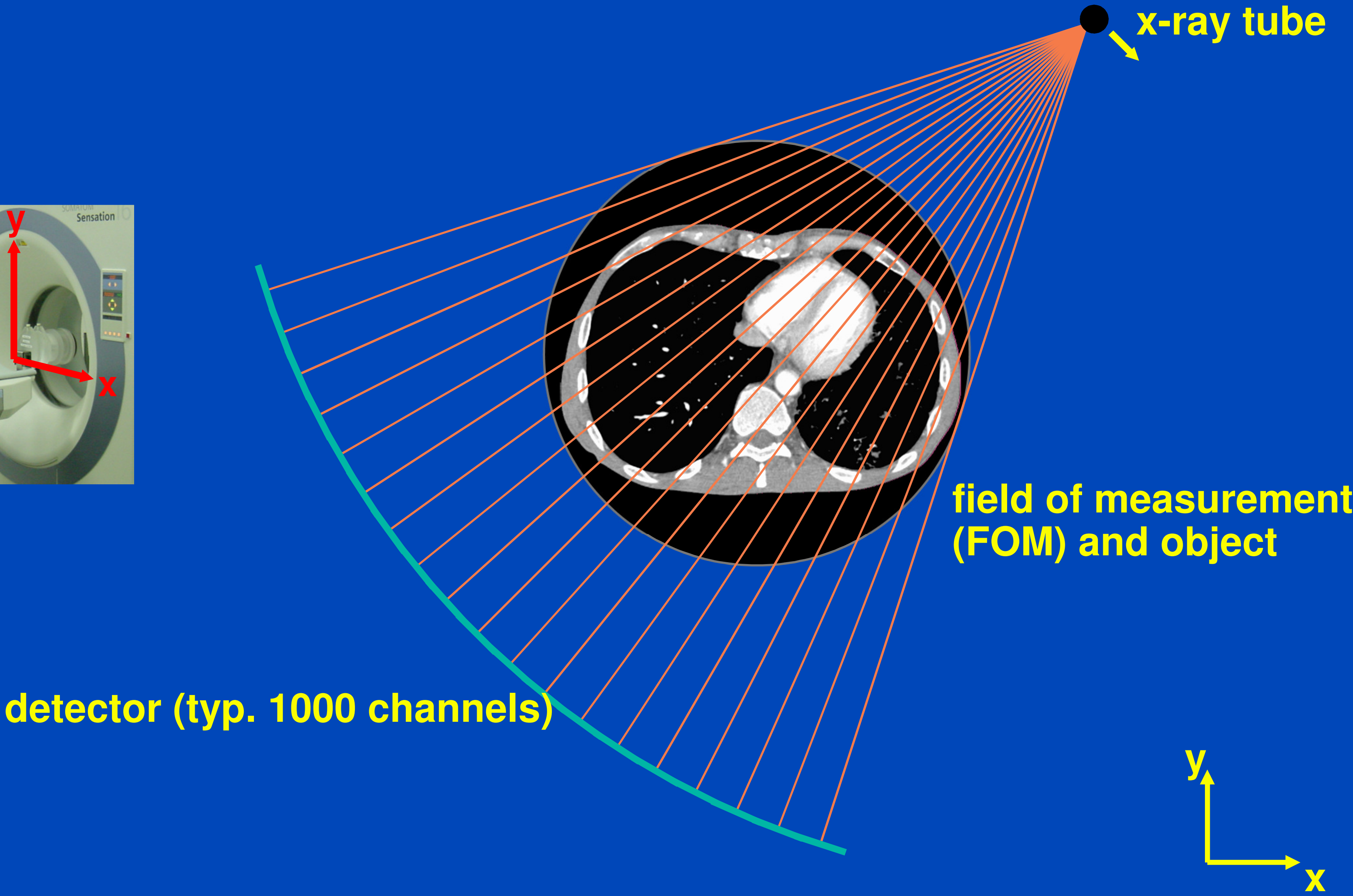
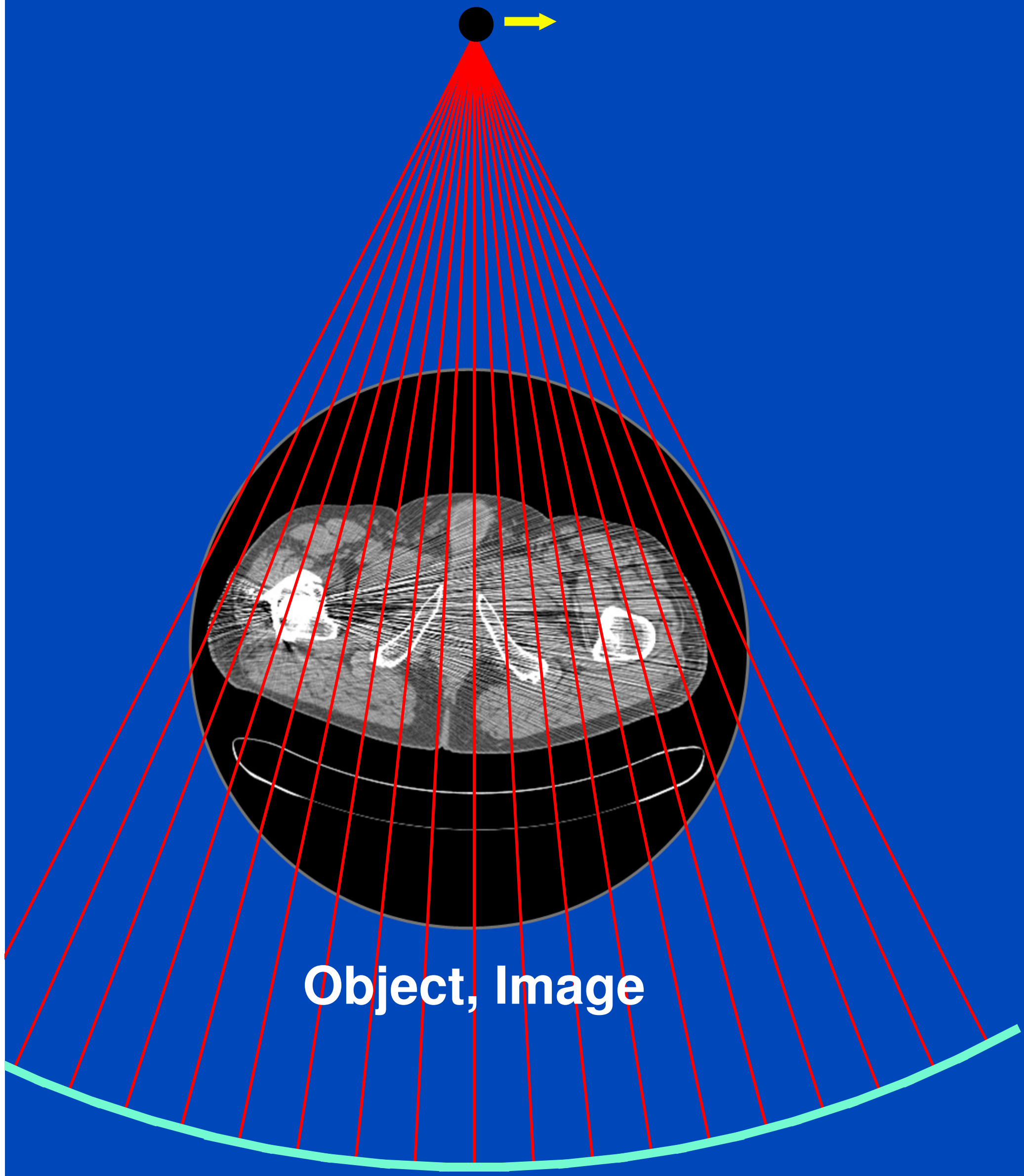
FRIEDRICH-ALEXANDER-UNIVERSITÄT ERLANGEN-NÜRNBERG

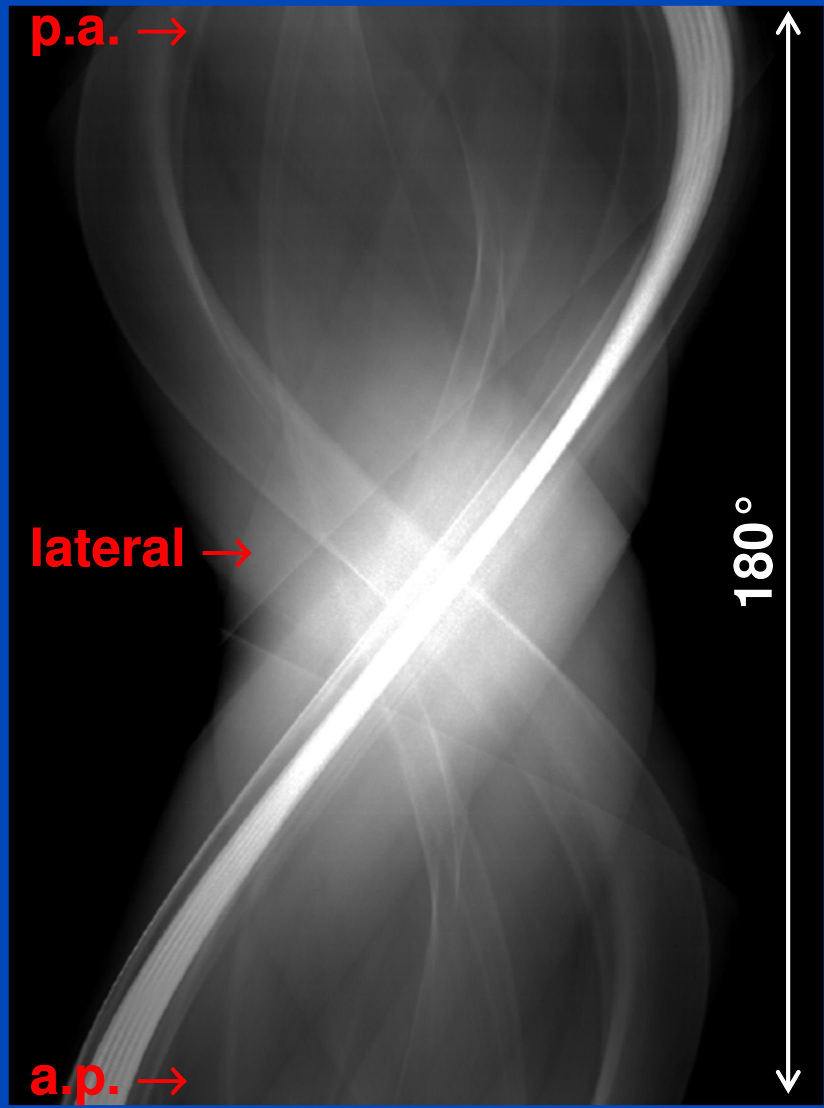
DEUTSCHES KREBSFORSCHUNGSZENTRUM IN DER HELMHOLTZ-GEMEINSCHAFT

Fan-Beam Geometry (transaxial / in-plane / x-y-plane)

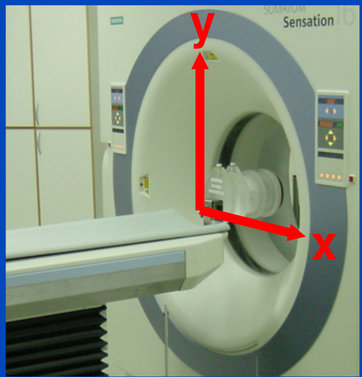


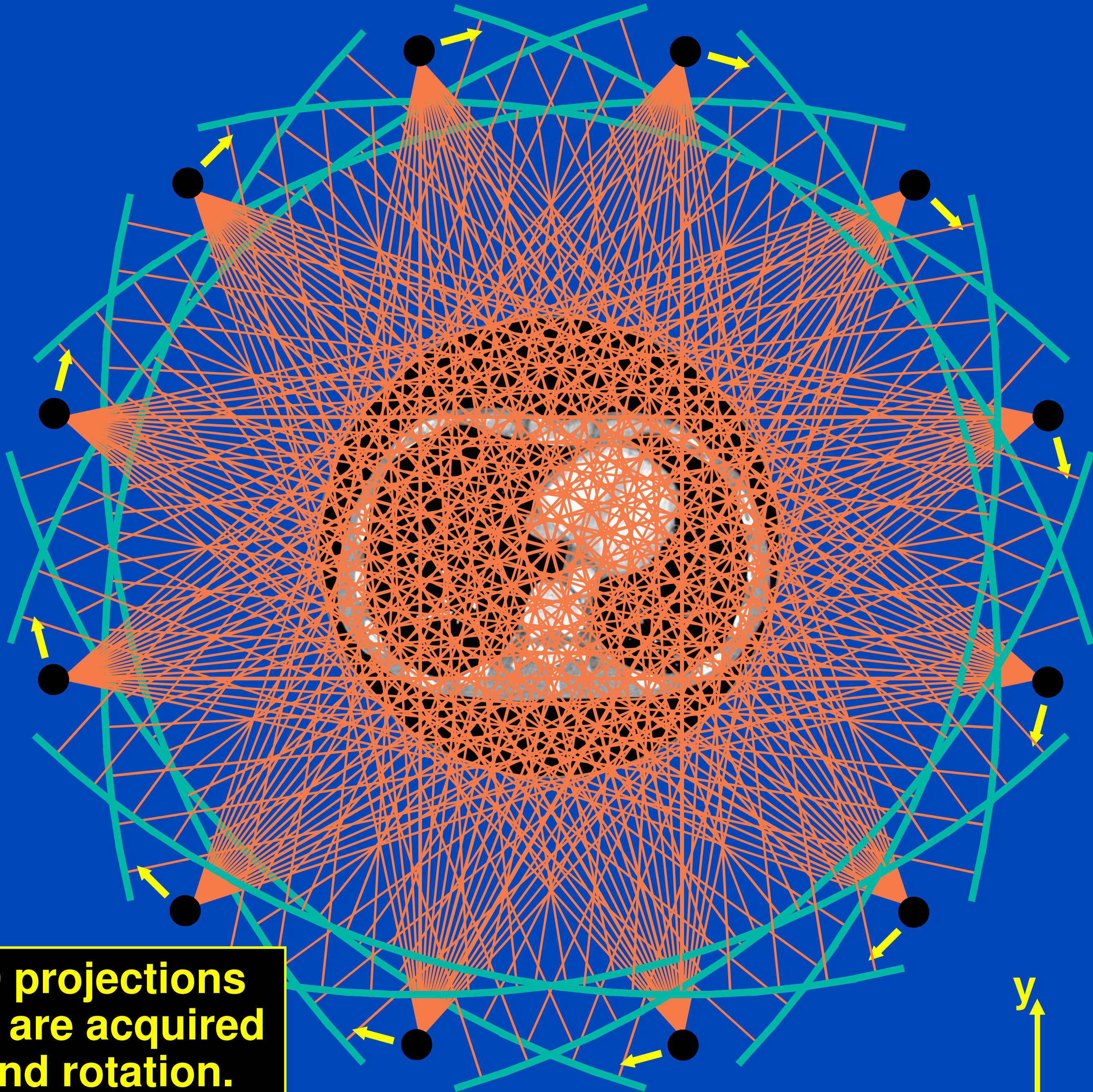




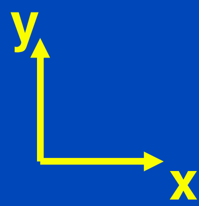


Sinogram, Rawdata

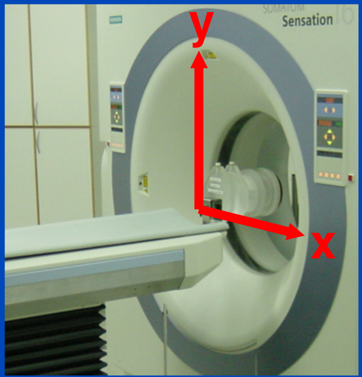


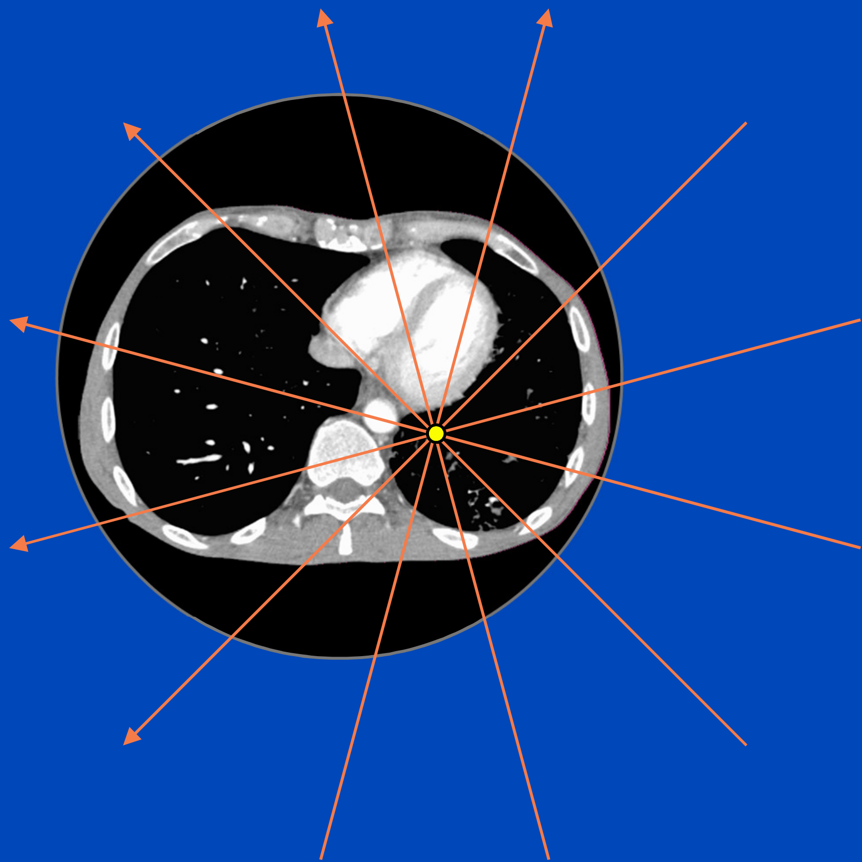


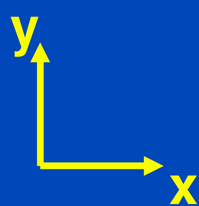
In the order of 1000 projections with 1000 channels are acquired per detector slice and rotation.



Data Completeness

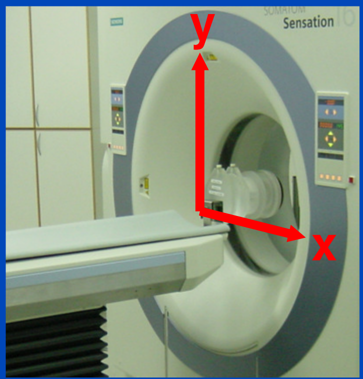


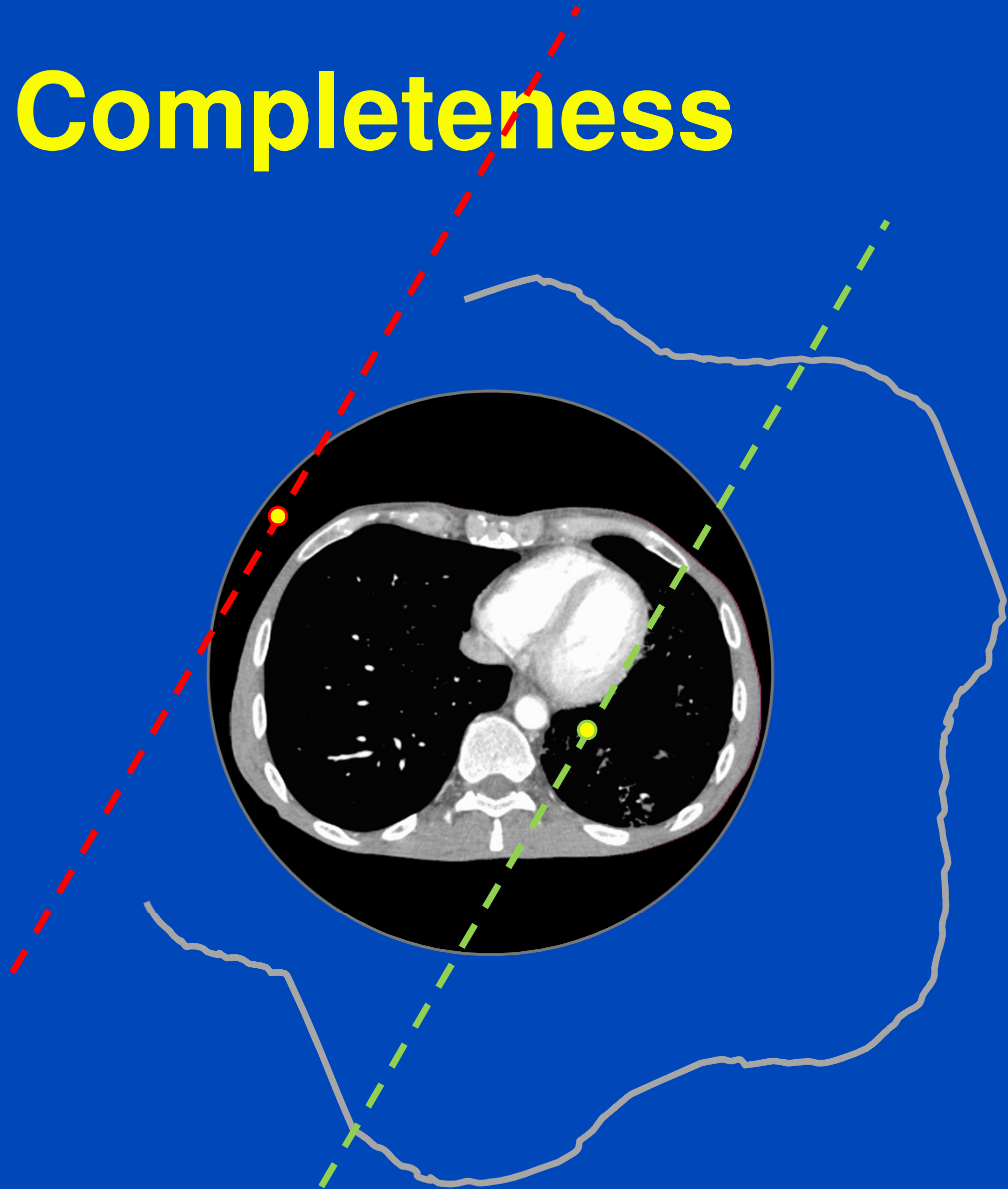




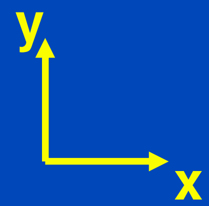
Each object point must be viewed by an angular interval of 180° or more. Otherwise image reconstruction is not possible.

Data Completeness





Any straight line through a voxel must be intersected by the source trajectory at least once.



Analytical Image Reconstruction

$$x^2 = y$$

Model

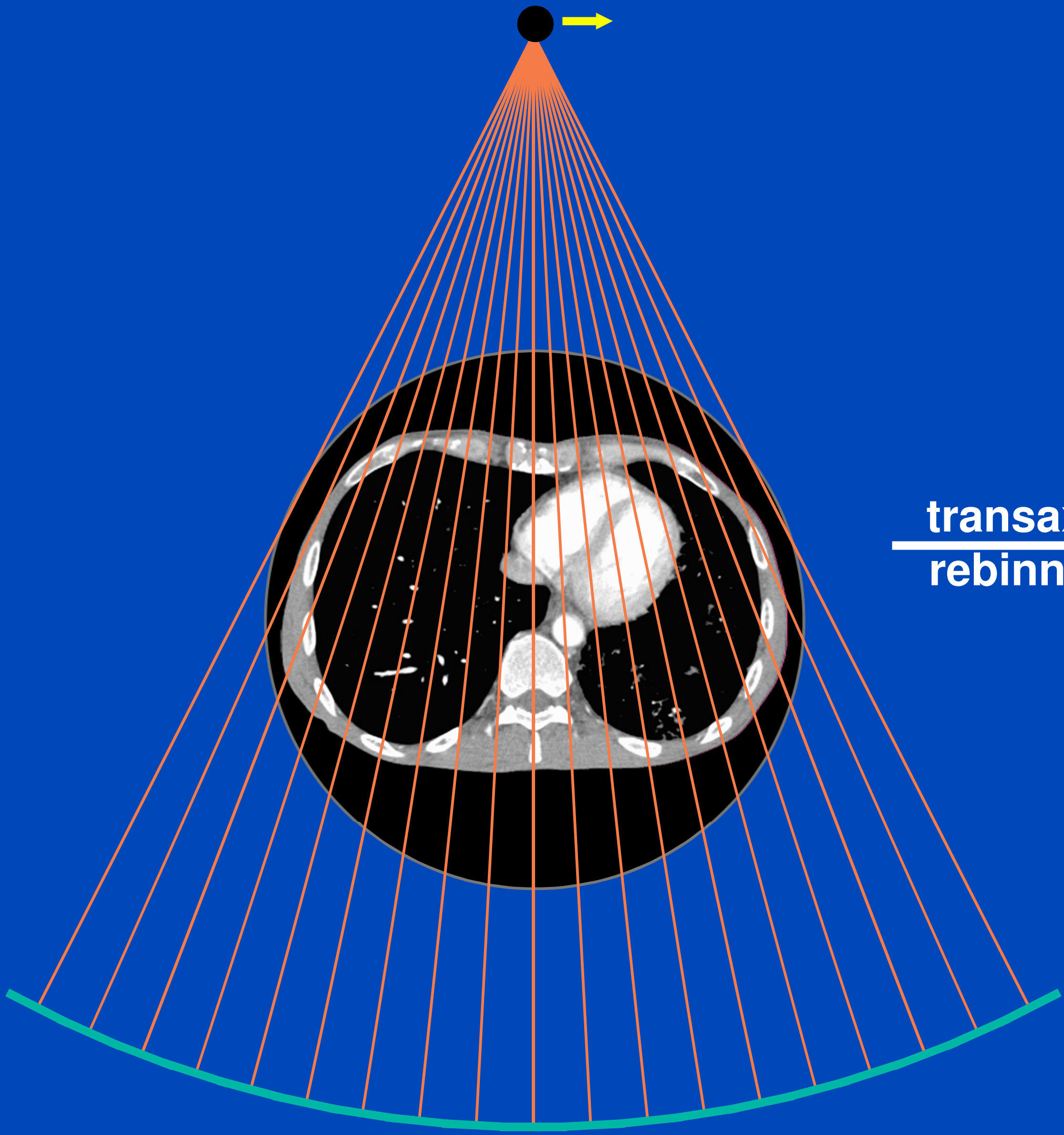
$$x = \sqrt{y}$$

Solution

2D: In-Plane Geometry

- Decouples from longitudinal geometry in many cases
- Useful for many imaging tasks
- Easy to understand
- 2D reconstruction options
  - Rebinning (resampling, resorting) to parallel beam geometry, followed by filtered backprojection (FBP)
  - Rebinning to parallel beam geometry followed by Fourier inversion
  - Filtered backprojection in the native fan-beam geometry
  - Backprojection filtration (BPF)
  - Expansion methods (representation of rawdata and image as series of orthogonal functions)

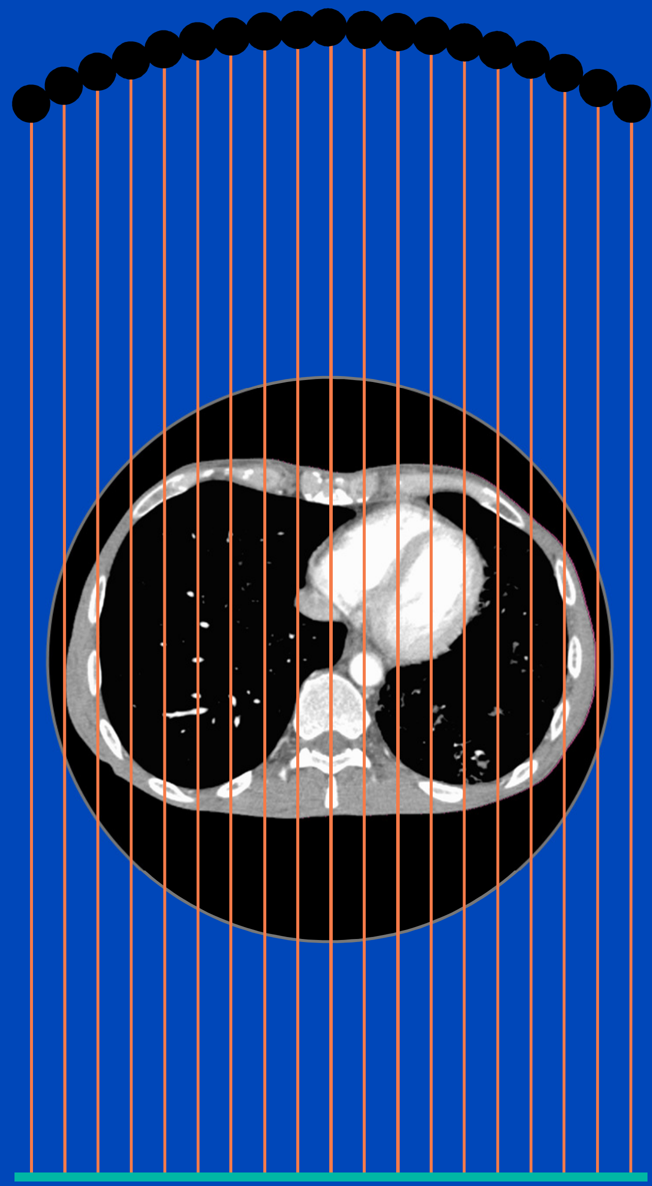
Fan-beam geometry



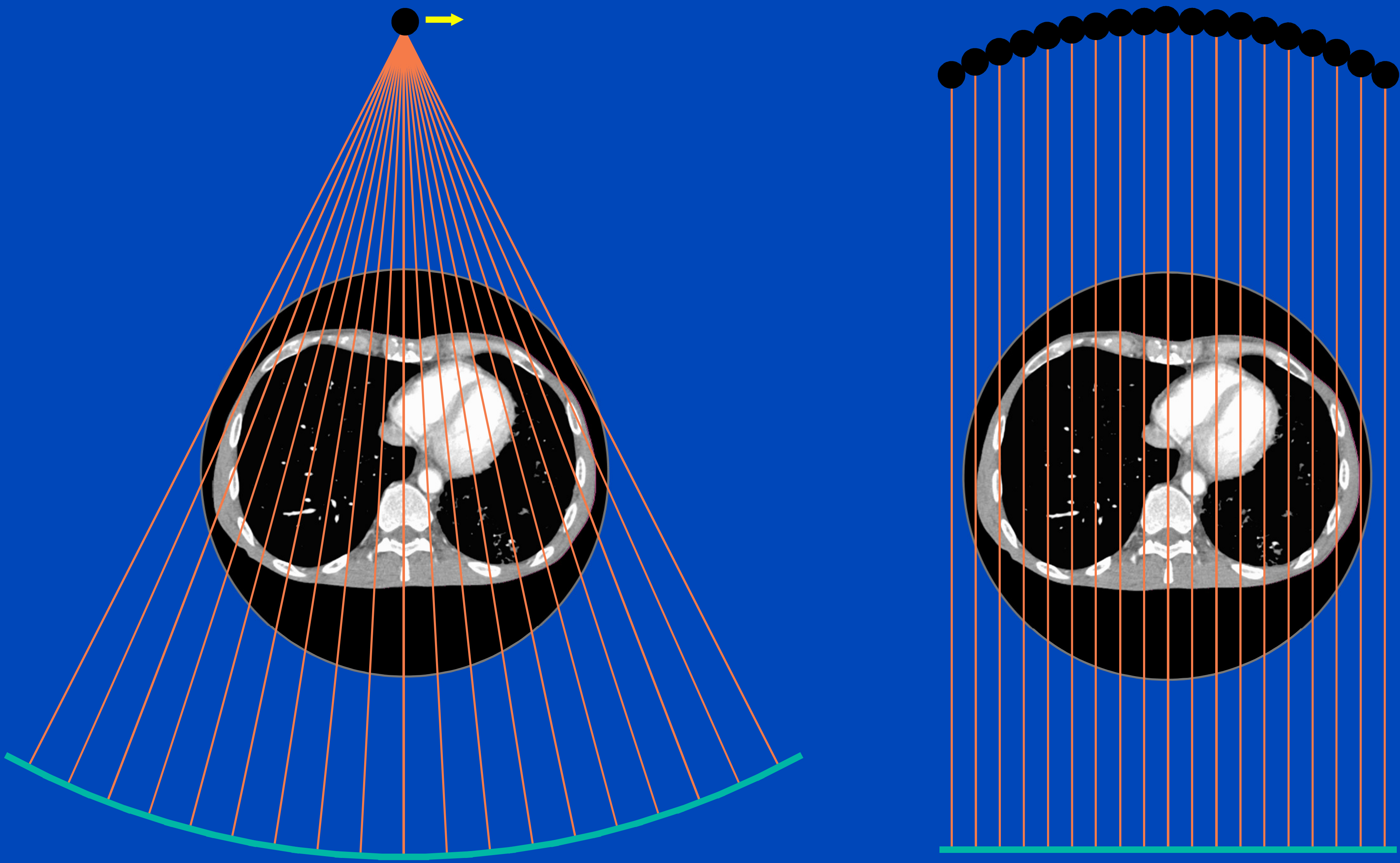
(α, β)

transaxial rebinning →

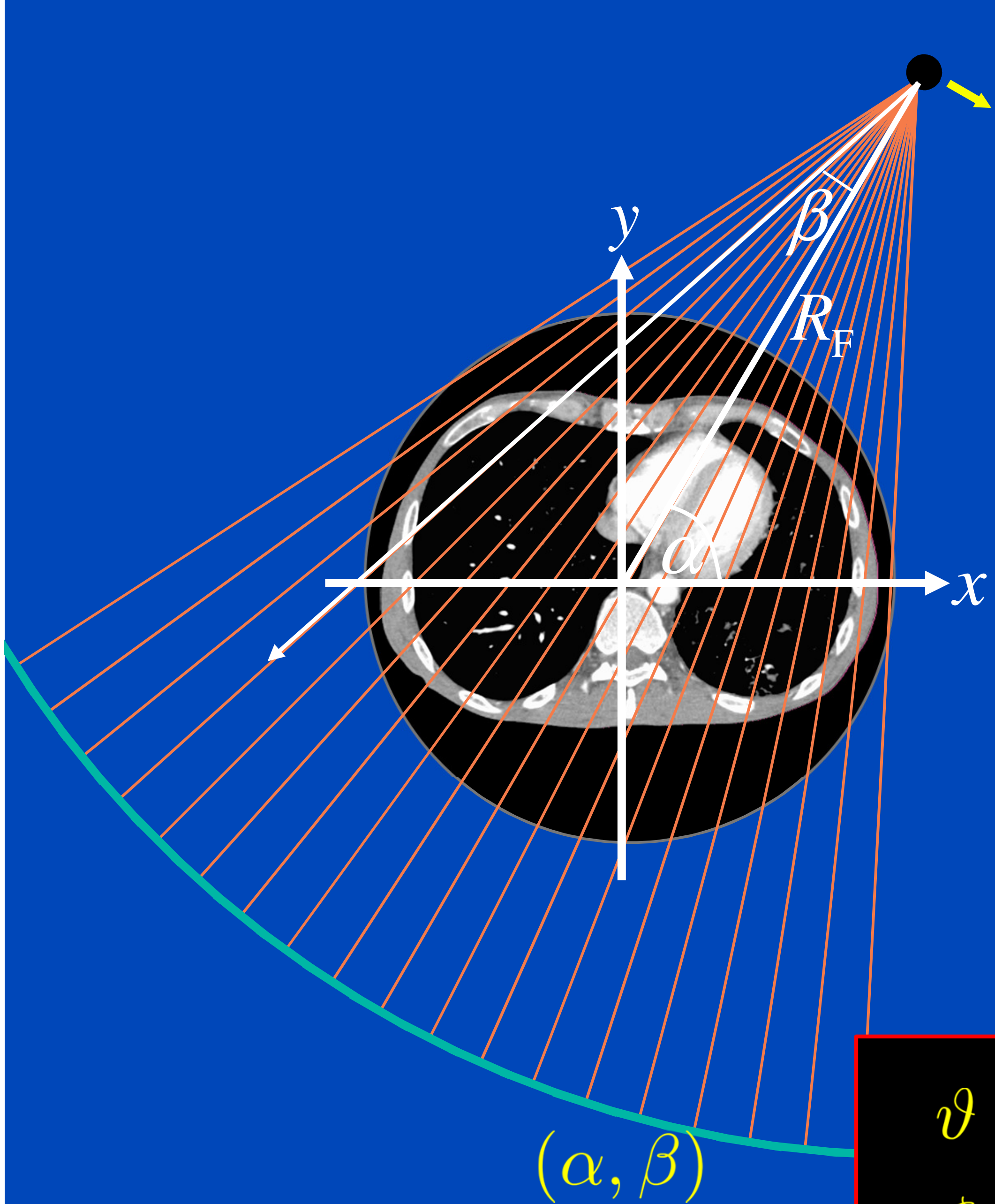
Parallel-beam geometry



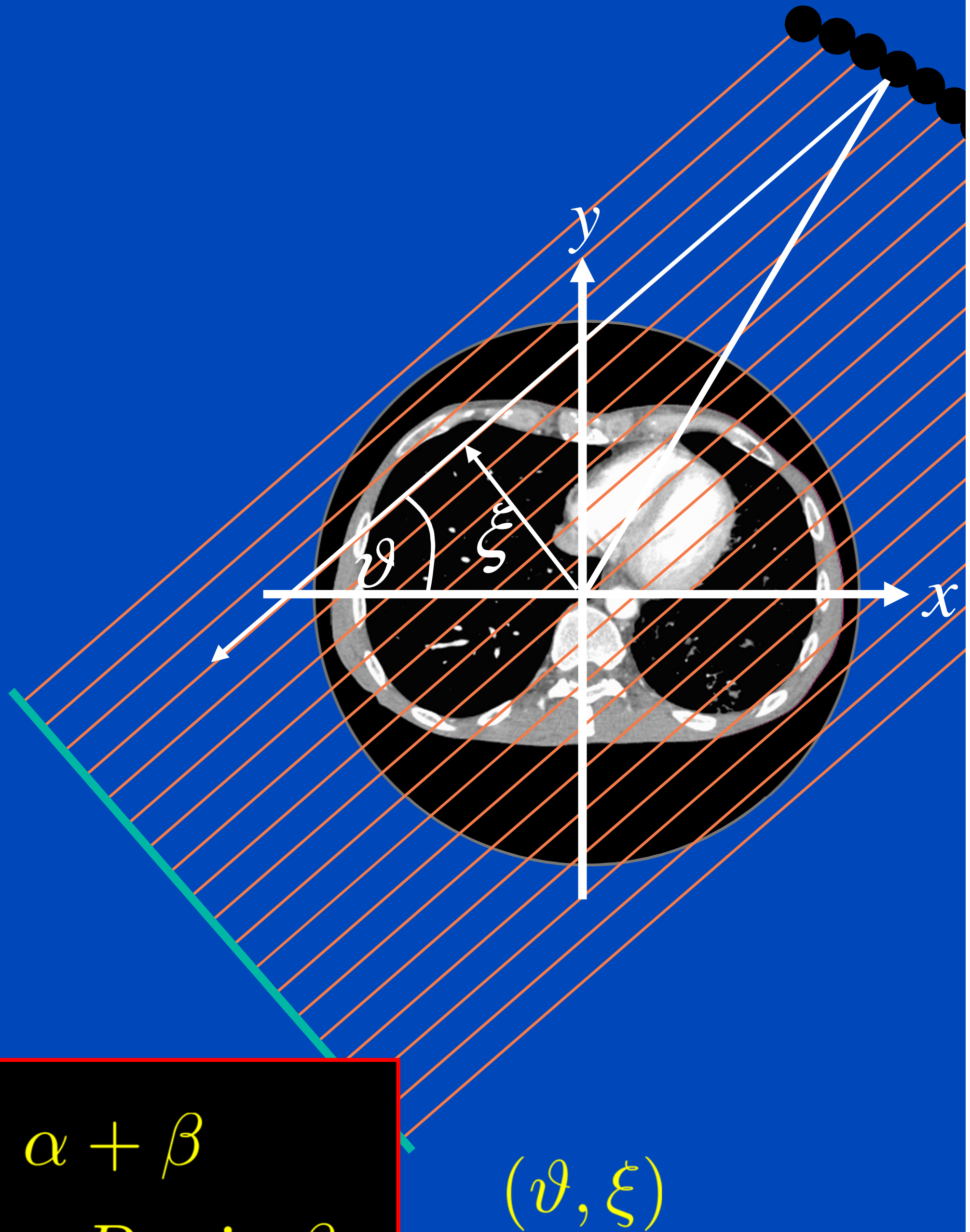
(ϑ, ξ)



Fan-beam geometry



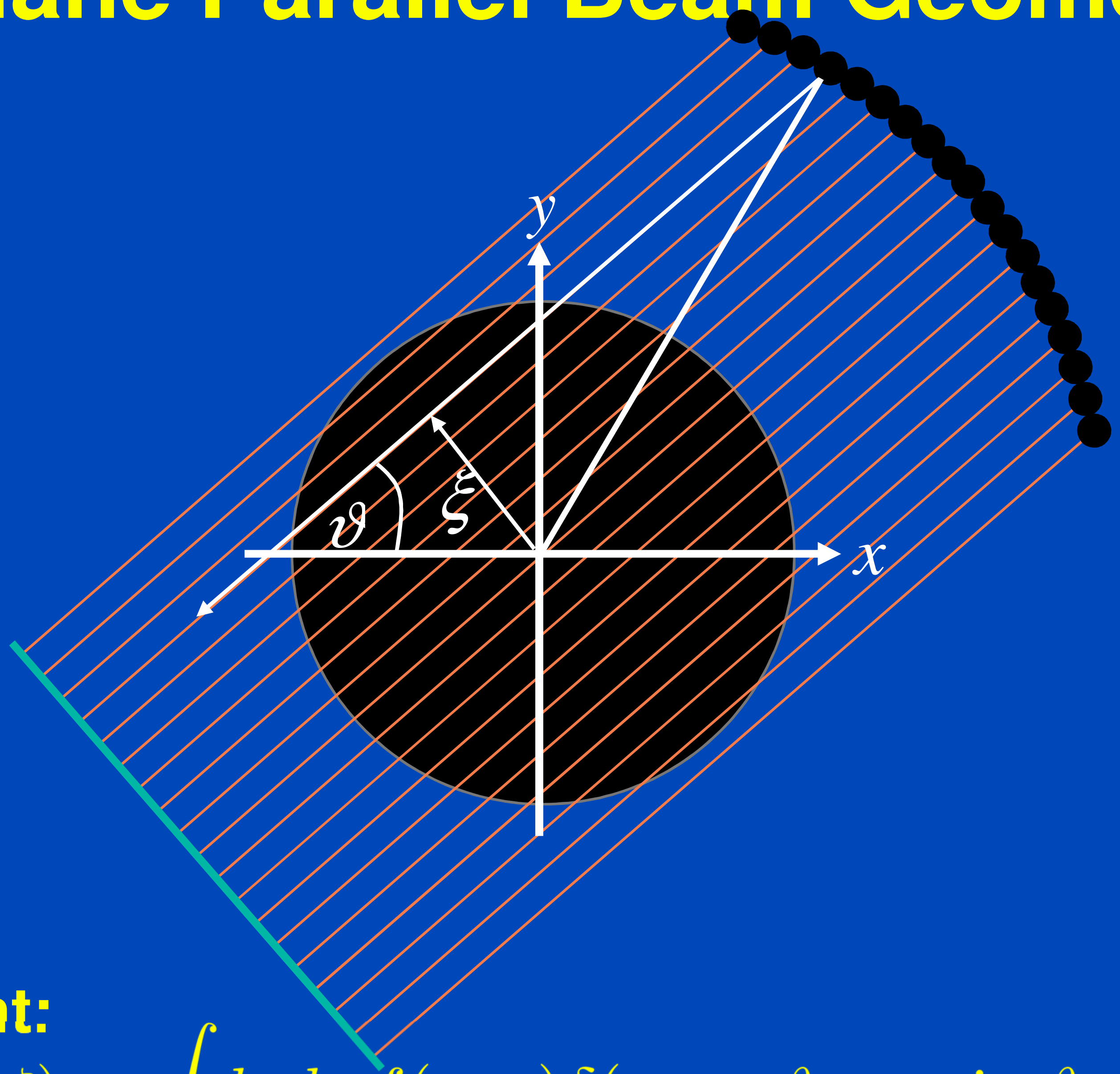
Parallel-beam geometry



$$\vartheta = \alpha + \beta$$

$$\xi = -R_{\mathrm{F}} \sin\beta$$

In-Plane Parallel Beam Geometry



Measurement:

$$p(\vartheta,\xi) = \int dx dy\, f(x,y)\delta(x\cos\vartheta + y\sin\vartheta - \xi)$$

Filtered Backprojection[1] (FBP)

Measurement: $p(\vartheta,\xi) = \int dxdy\, f(x,y)\delta(x\cos\vartheta + y\sin\vartheta - \xi)$

Fourier transform:

$$\int d\xi\, p(\vartheta,\xi)e^{-2\pi i\xi u} = \int dxdy\, f(x,y)e^{-2\pi iu(x\cos\vartheta + y\sin\vartheta)}$$

This is the central slice theorem: $P(\vartheta,u) = F(u\cos\vartheta, u\sin\vartheta)$

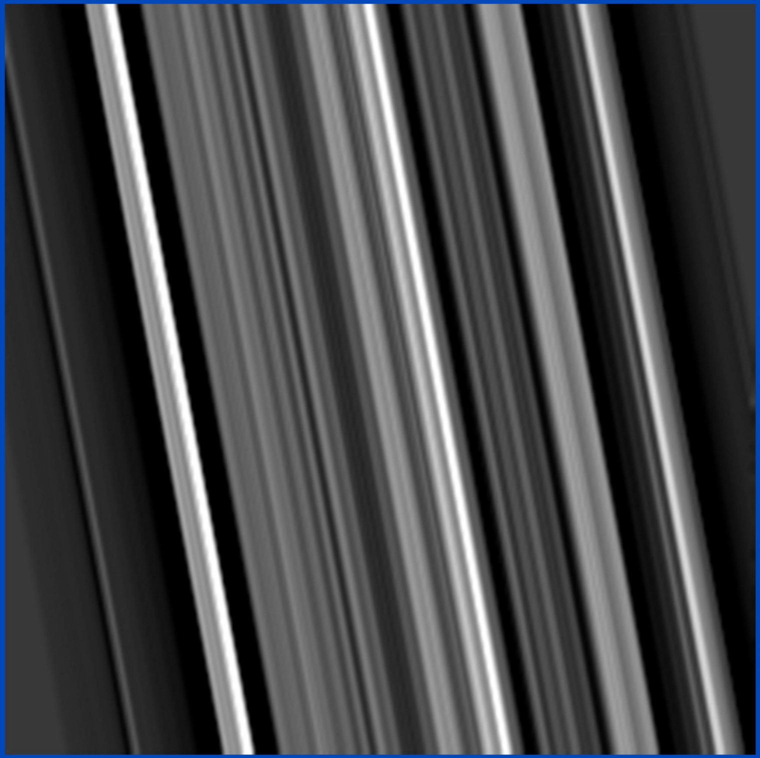
Inversion: $f(x,y) = \int_0^\pi d\vartheta \int_{-\infty}^{\infty} du\, |u| P(\vartheta,u)e^{2\pi iu(x\cos\vartheta + y\sin\vartheta)}$

$$= \int_0^\pi d\vartheta\, p(\vartheta,\xi) * k(\xi)\Big|_{\xi = x\cos\vartheta + y\sin\vartheta}$$

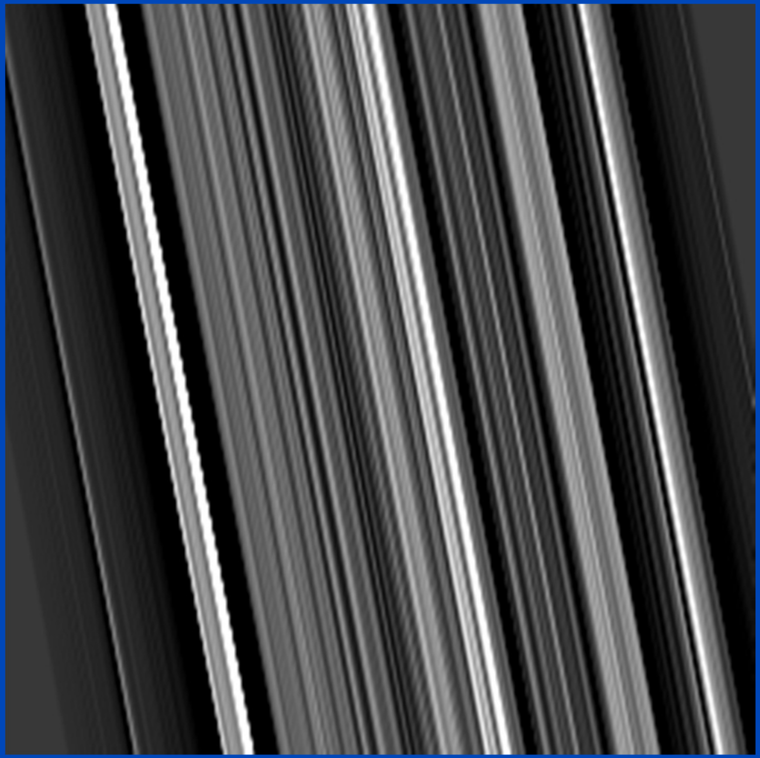
[1]Ramachandran and Lakshminarayanan. Proc. Nat. Acad. Sci. USA, 1971.

Filtered Backprojection (FBP)

1. Filter projection data with the reconstruction kernel.
2. Backproject the filtered data into the image:

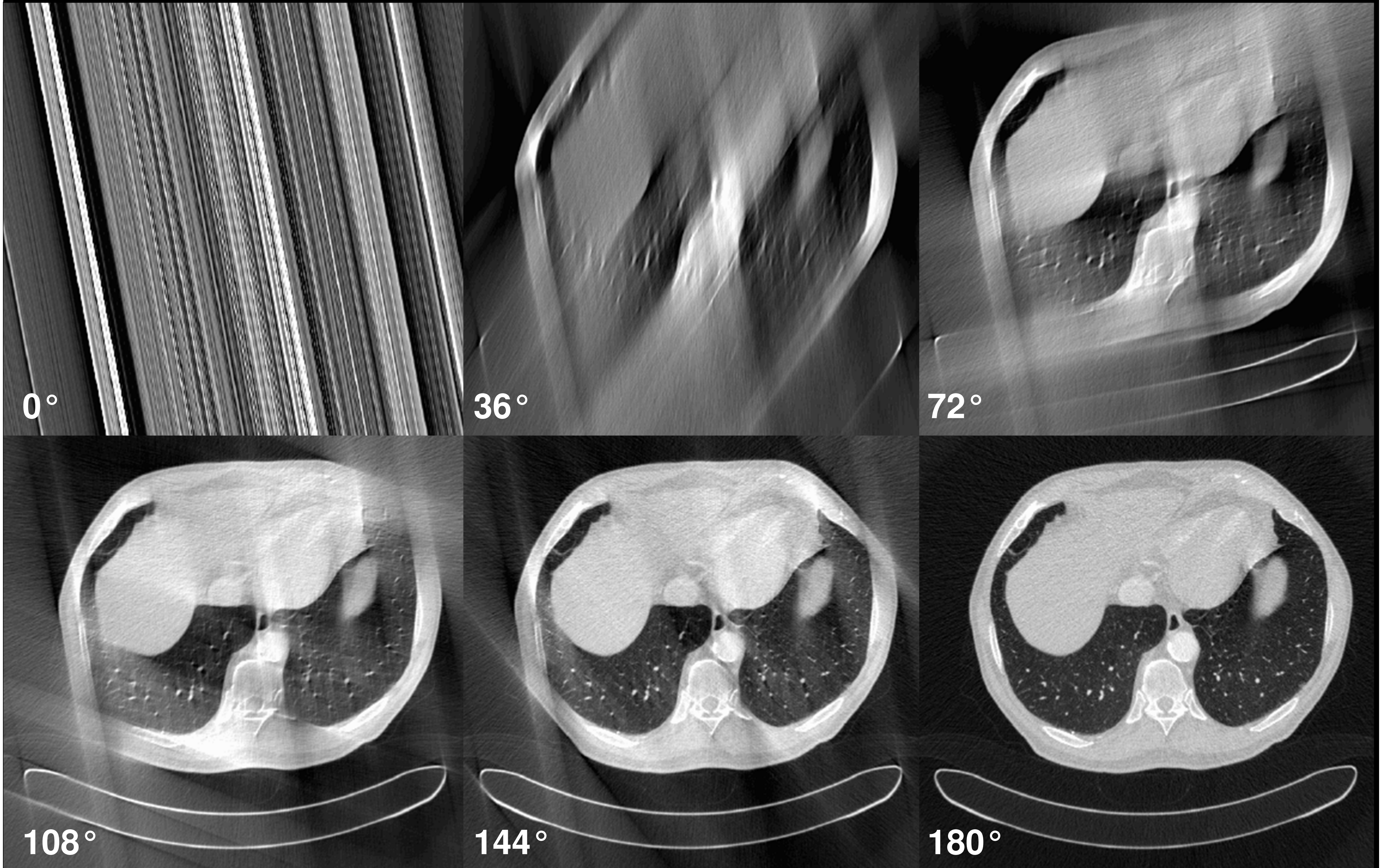


Smooth



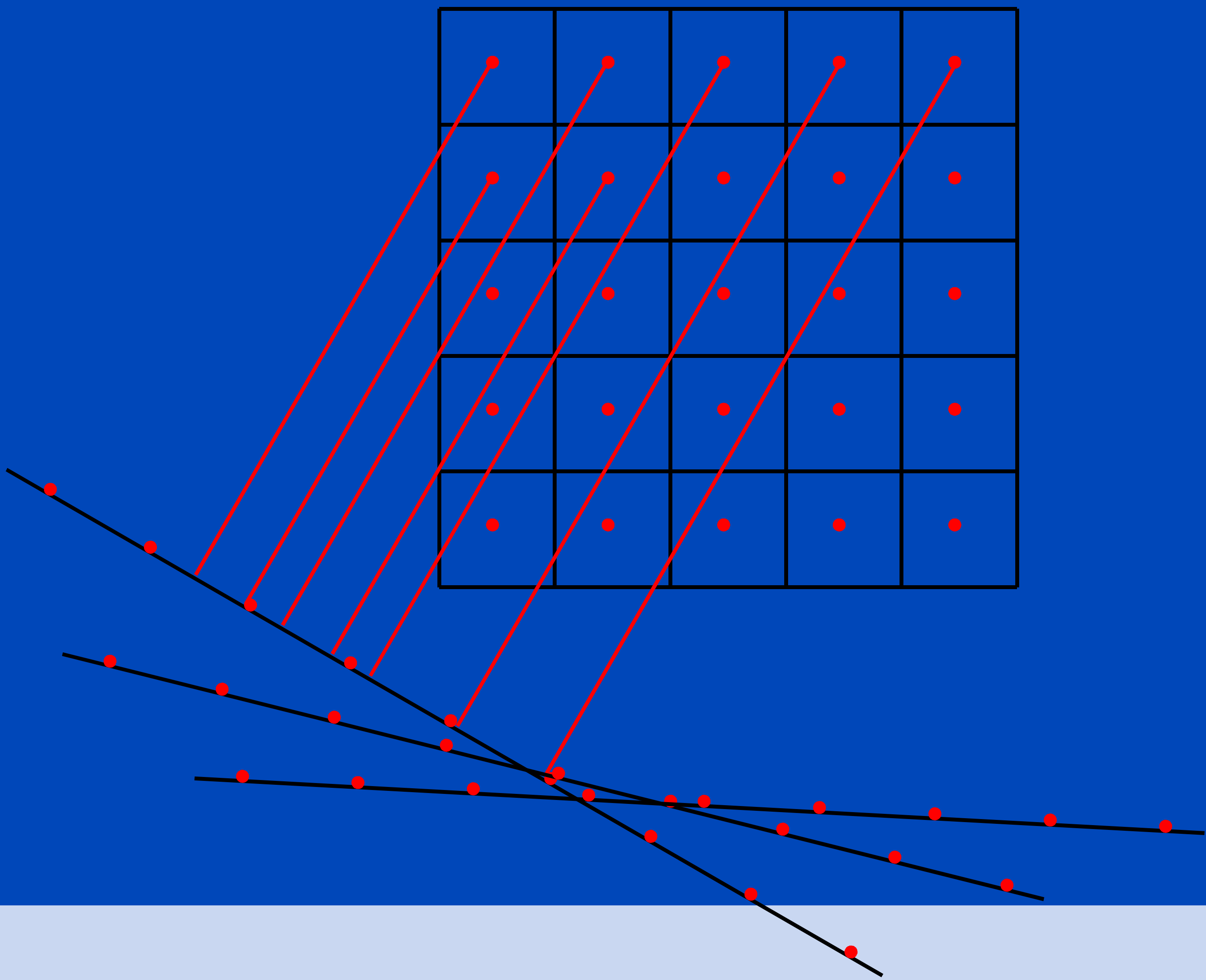
Standard

Reconstruction kernels balance between spatial resolution and image noise.

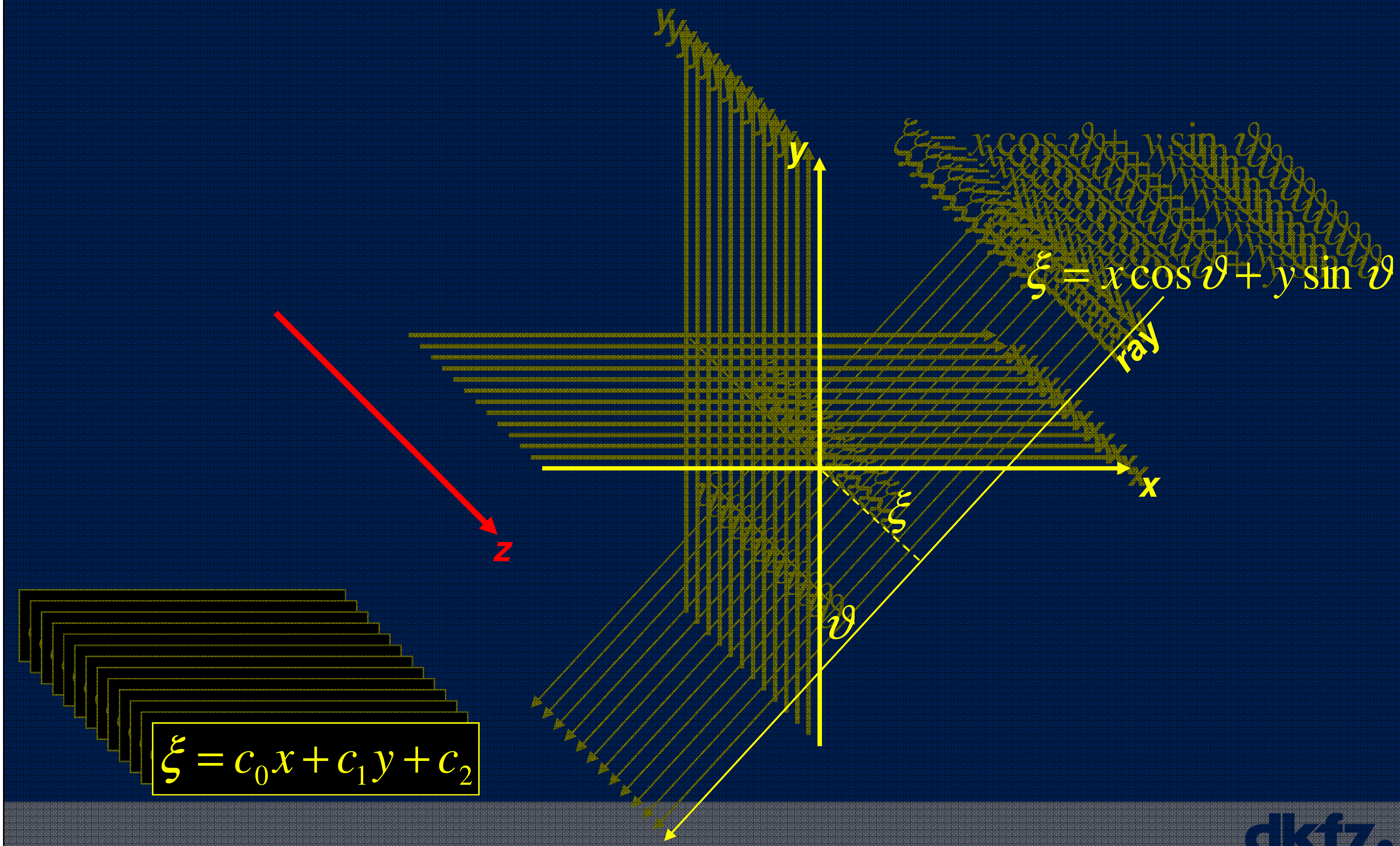


dkfz.

Backprojection



Parallel-Beam Geometry



Parallel Backprojection: Geometry

voxel position

projection data

slice number or position

$$f(\boldsymbol{r}) = \int d\vartheta \; p(\vartheta, \xi(\vartheta, \boldsymbol{r}), z)$$

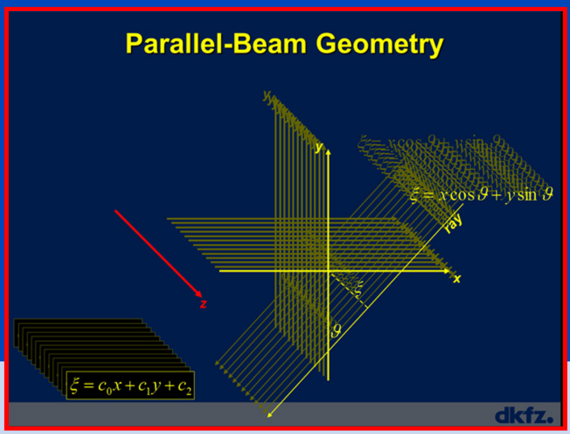
reconstructed slices

$$\xi(\vartheta, \boldsymbol{r}) = c_0 x + c_1 y + c_2 \qquad c_i = c_i(\vartheta)$$

trajectory parameter

distance of ray to origin

transform coefficients

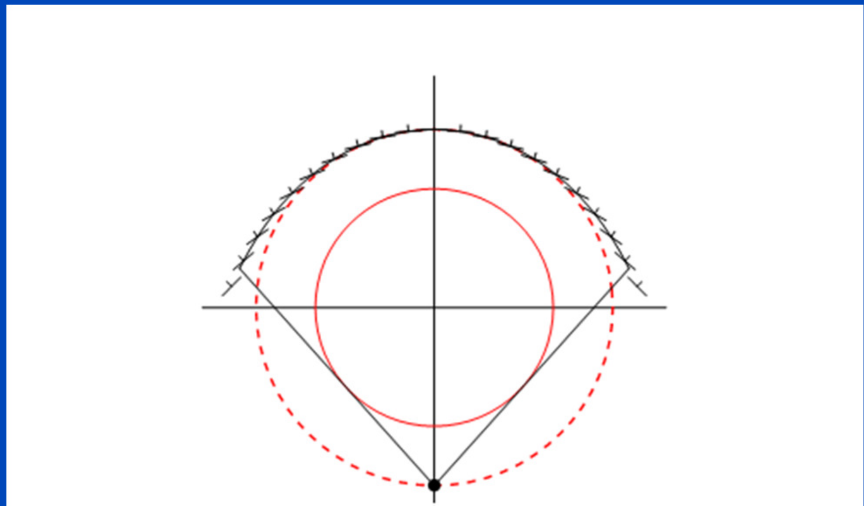


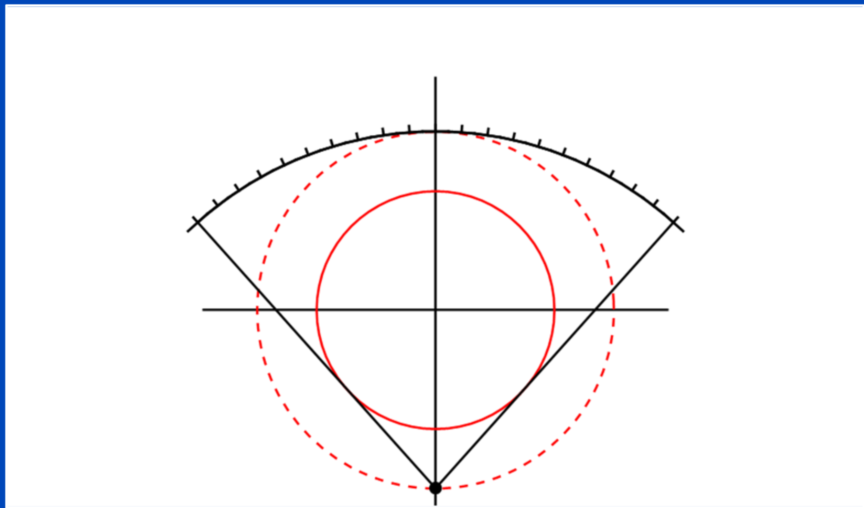
Parallel Backprojection: Reference Implementation

```
void ParBackProjRefLI(float       * const Vol, int const I, int const J, int const K,
                      float const * const Raw, int const N, int const M,
                      float const * const c0, ..., float const * const c2)
    {
    for(int n=0; n<N; n++) // projection index (theta)
    for(int i=0; i<I; i++) // slow voxel index (x)
    for(int j=0; j<J; j++) // med. voxel index (y)
        {
        float const mreal=c0[n]*i+c1[n]*j+c2[n]; // detector channel index (xi)
          int const m =int(mreal); // lower sample position
        float const wm=mreal-m; // linear interpolation weight

        for(int k=0; k<K; k++) // fast voxel and detector row index (z)
            {
            #define V(i, j, k) Vol[((i)*J+j)*K+k] // linear memory layout, use V and
            #define R(n, m, k) Raw[((n)*M+m)*K+k] // R as shortcuts for Vol and Raw

            V(i, j, k)+=(1-wm)*R(n, m, k)+wm*R(n, m+1, k);

            #undef V
            #undef R
            }
        }
    }
```

2D Fan-Beam FBP

- Some fan-beam geometries lend themselved to filtered backprojection without rebinning[1].
- Among those geometries the geometry with equiangular sampling in β, i.e. in steps of $\Delta\beta$, is the most prominent one (although not necessarily optimal).
- The second most prominent geometry that allows for filtered backprojection in the native geometry is the one corresponding to a flat detector.
- The fourth generation CT geometry does not allow for shift-invariant filtering, unless the distance R_F of the focal spot to the isocenter equals the radius R_D of the detector ring.





[1]Guy Besson. CT fan-beam parametrizations leading to shift-invariant filtering. Inv. Prob. 1996.

2D Fan-Beam FBP

- **Classical way (coordinate transform):**

$$f(\boldsymbol{r}) = \tfrac{1}{2}\int_0^{2\pi} d\alpha \frac{1}{|\boldsymbol{r}-\boldsymbol{s}(\alpha)|^2} \; R_\mathrm{F}\cos\beta\, q(\alpha,\beta) \,*\, k(\sin\beta)\Big|_{\beta=\hat\beta(\alpha,\boldsymbol{r})}$$

- **Modern way[1] (inspired by Katsevich's work):**

$$f(\boldsymbol{r}) = \tfrac{1}{2}\int_0^{2\pi} d\alpha \frac{1}{|\boldsymbol{r}-\boldsymbol{s}(\alpha)|} \,(\partial_\beta - \partial_\alpha)\, q(\alpha,\beta) * K(\sin\beta)\Big|_{\beta=\hat\beta(\alpha,\boldsymbol{r})}$$

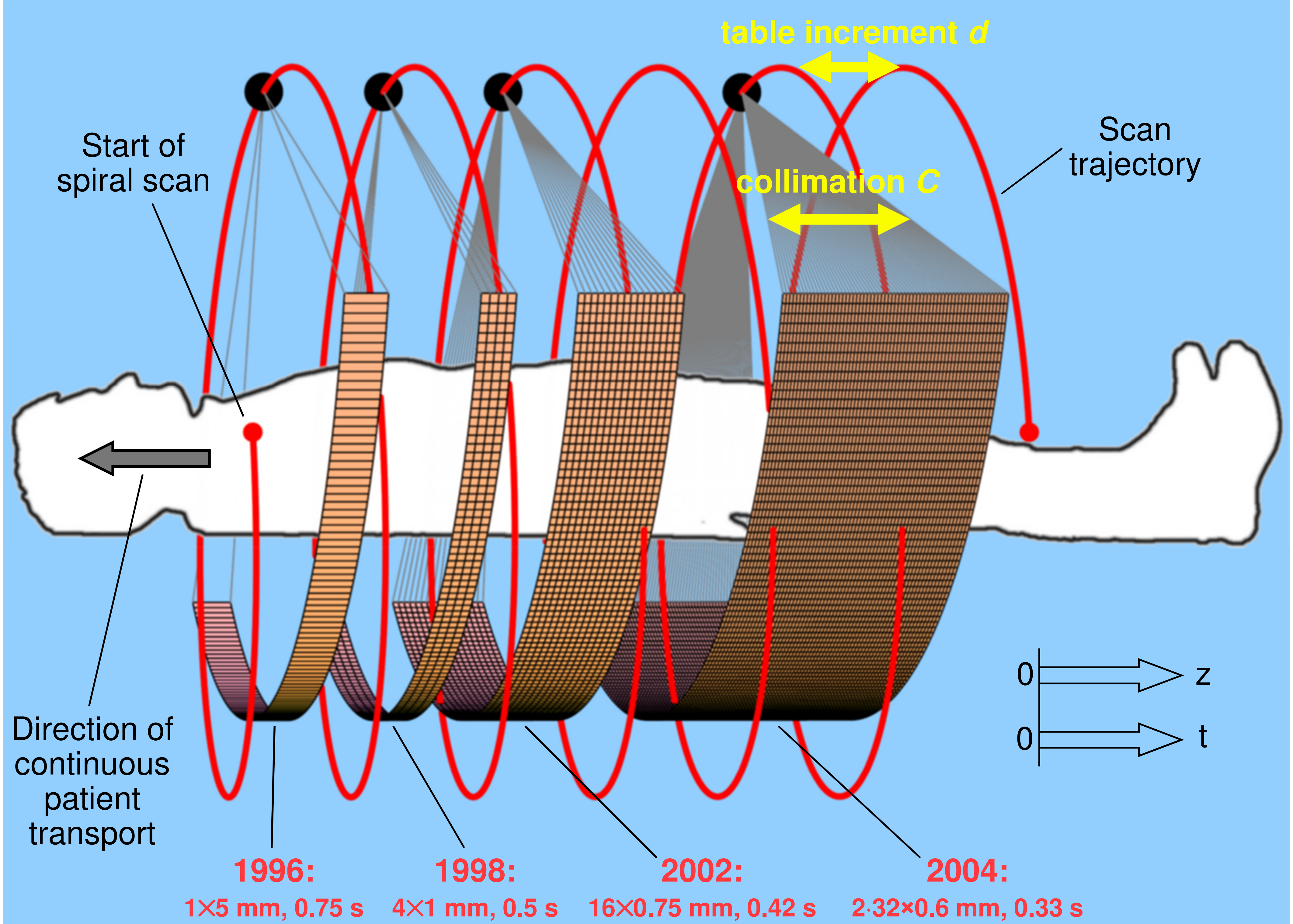
- **Parallel beam FBP for comparison:**

$$f(\boldsymbol{r}) = \tfrac{1}{2}\int_0^{2\pi} d\vartheta\, p(\vartheta,\xi) * k(\xi)\Big|_{\xi=\hat\xi(\vartheta,\boldsymbol{r})}$$

$$\hat\beta(\alpha,\boldsymbol{r}) = -\sin^{-1}\frac{x\cos\alpha + y\sin\alpha}{|\boldsymbol{r}-\boldsymbol{s}(\alpha)|}$$

$$\hat\xi(\vartheta,\boldsymbol{r}) = x\cos\vartheta + y\sin\vartheta$$

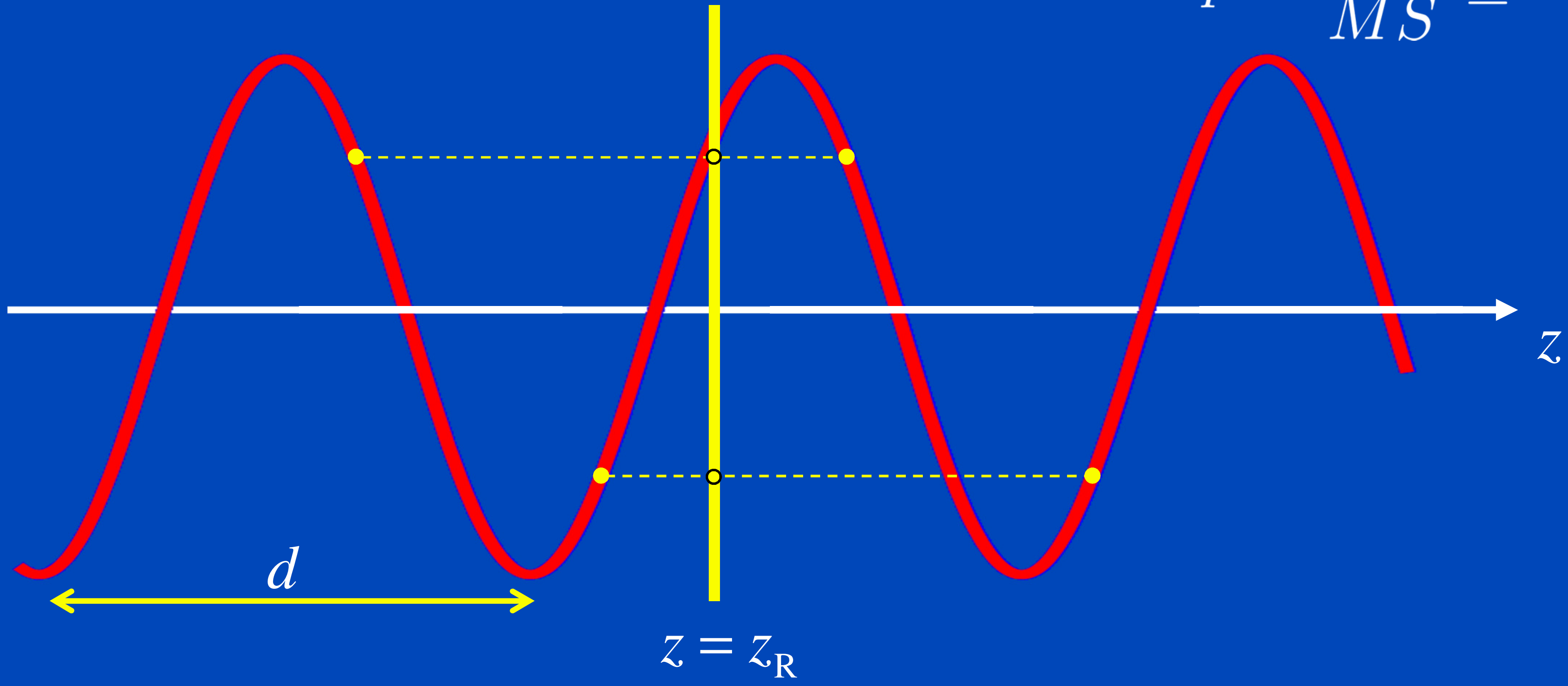
[1]F. Noo et al. Image reconstruction from fan-beam projections on less than a short scan. PMB 2002.



Kalender et al., Radiology 173(P):414 (1989) and 176:181-183 (1990)

360° LI Spiral z-Interpolation for Single-Slice CT (*M*=1)

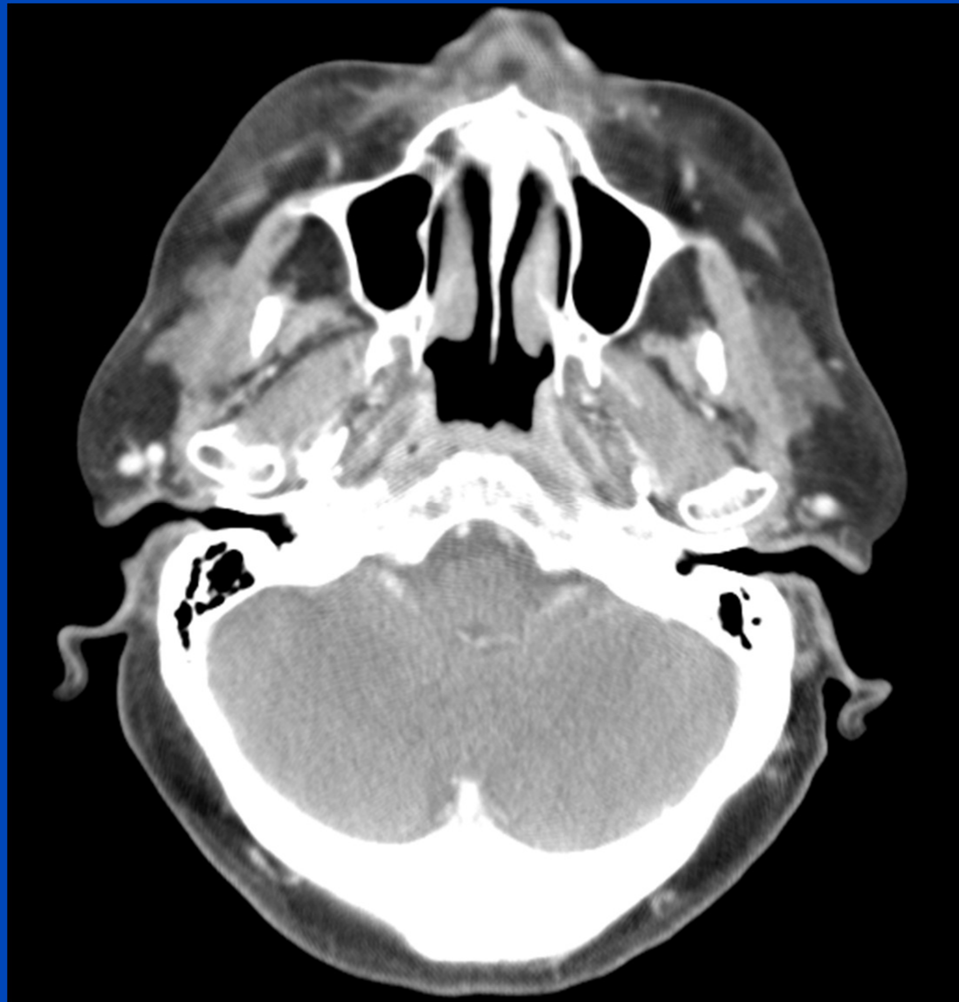
$$p = \frac{d}{MS} \leq 2$$



Spiral z-interpolation is typically a linear interpolation between points adjacent to the reconstruction position to obtain circular scan data.

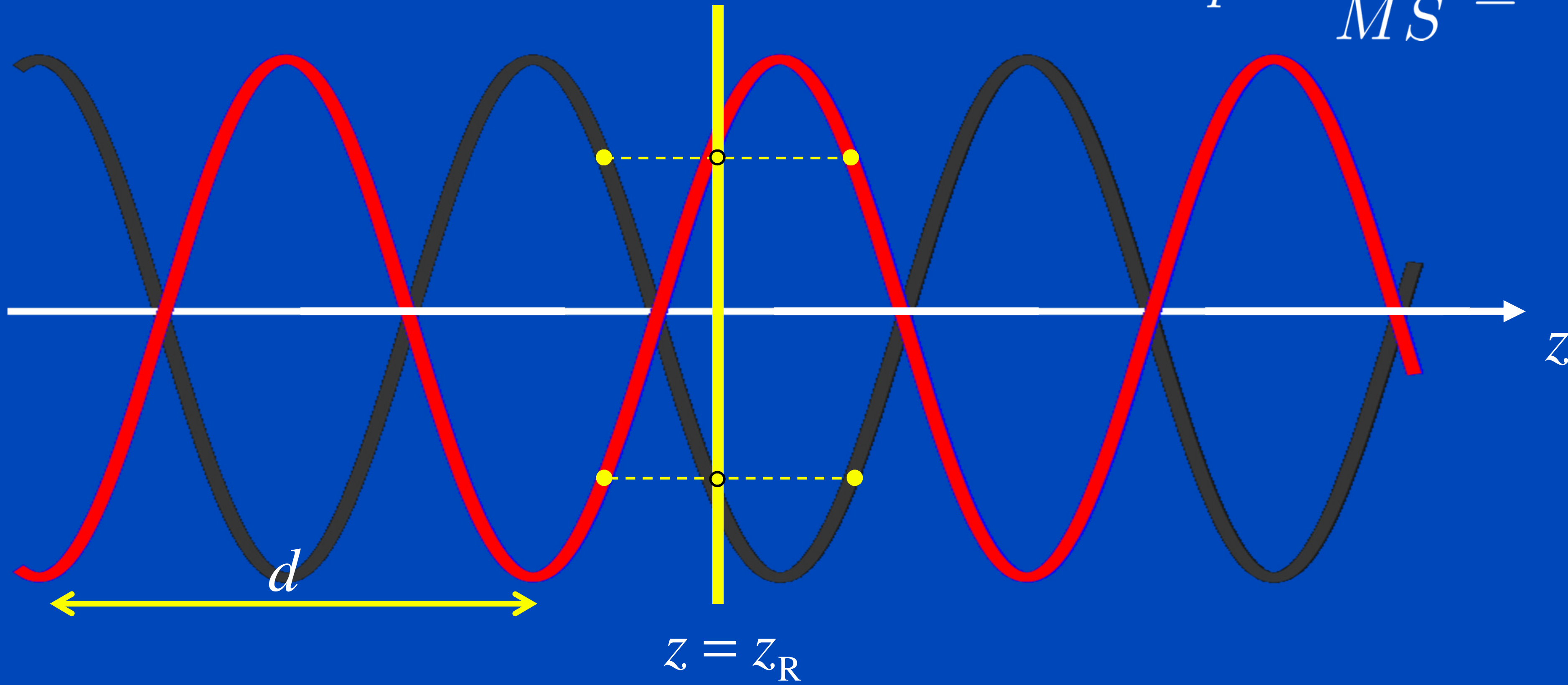
without *z*-interpolation

with *z*-interpolation



180° LI Spiral z-Interpolation for Single-Slice CT (*M*=1)

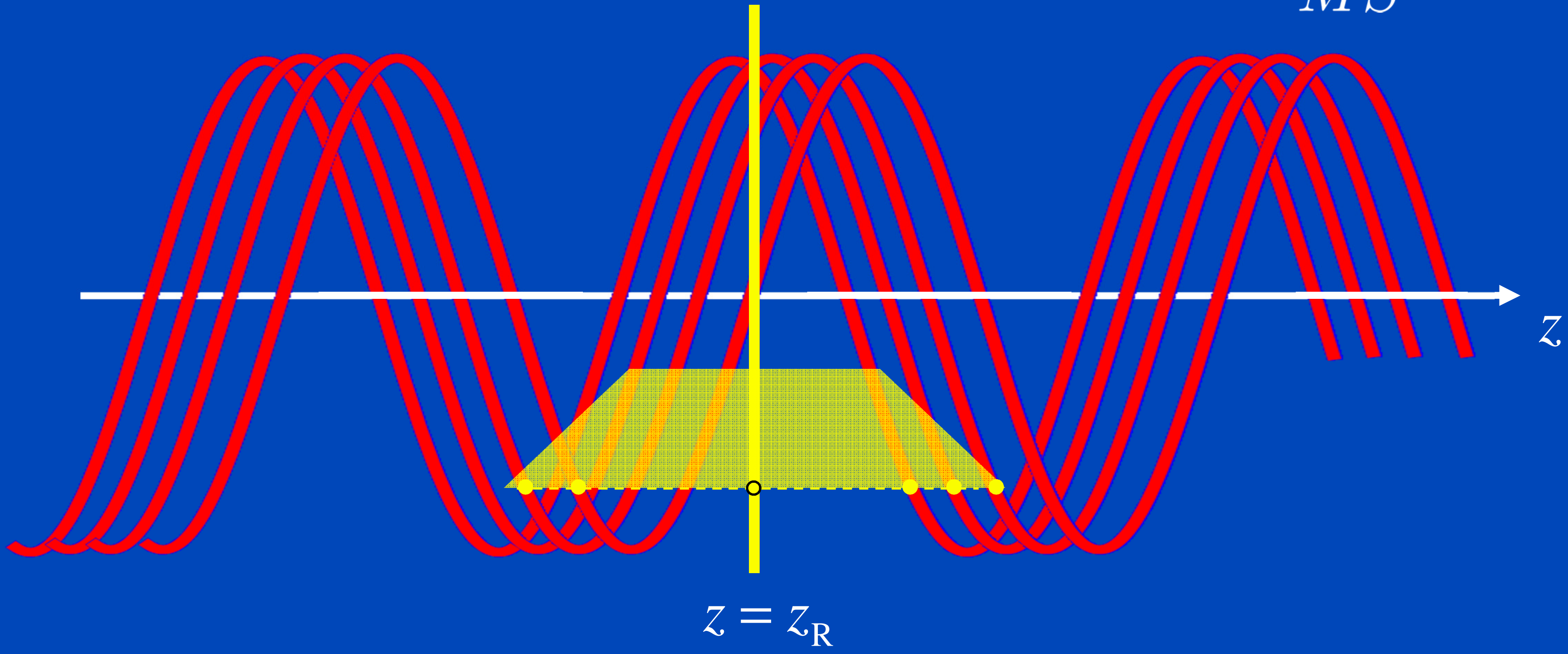
$$p = \frac{d}{MS} \leq 2$$



180° Spiral z-interpolation interpolates between direct and complementary rays.

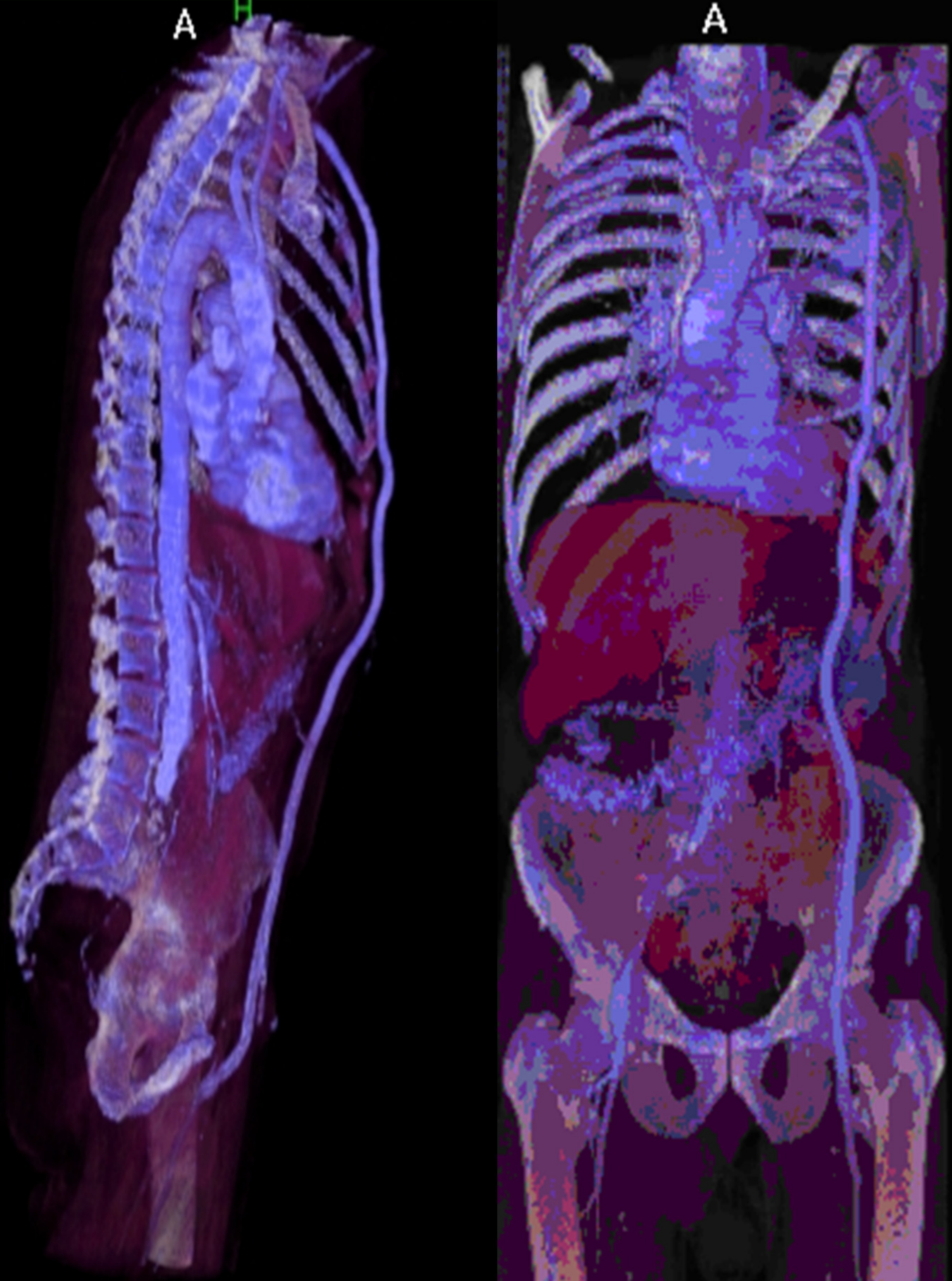
Spiral z-Filtering for Multi-Slice CT
M=2, …, 6

$$p = \frac{d}{MS} \le 1.5$$



Spiral z-filtering is collecting data points weighted with a triangular or trapezoidal distance weight to obtain circular scan data.

dkfz.



CT Angiography: Axillo-femoral bypass

***M* = 4**

120 cm in 40 s

0.5 s per rotation
4×2.5 mm collimation
pitch 1.5

The Pitch Value is the Measure for Scan Overlap

The pitch is defined as the ratio of the table increment per full rotation to the *total* collimation width in the center of rotation:

$$p = \frac{d}{C} = \frac{d}{MS}$$

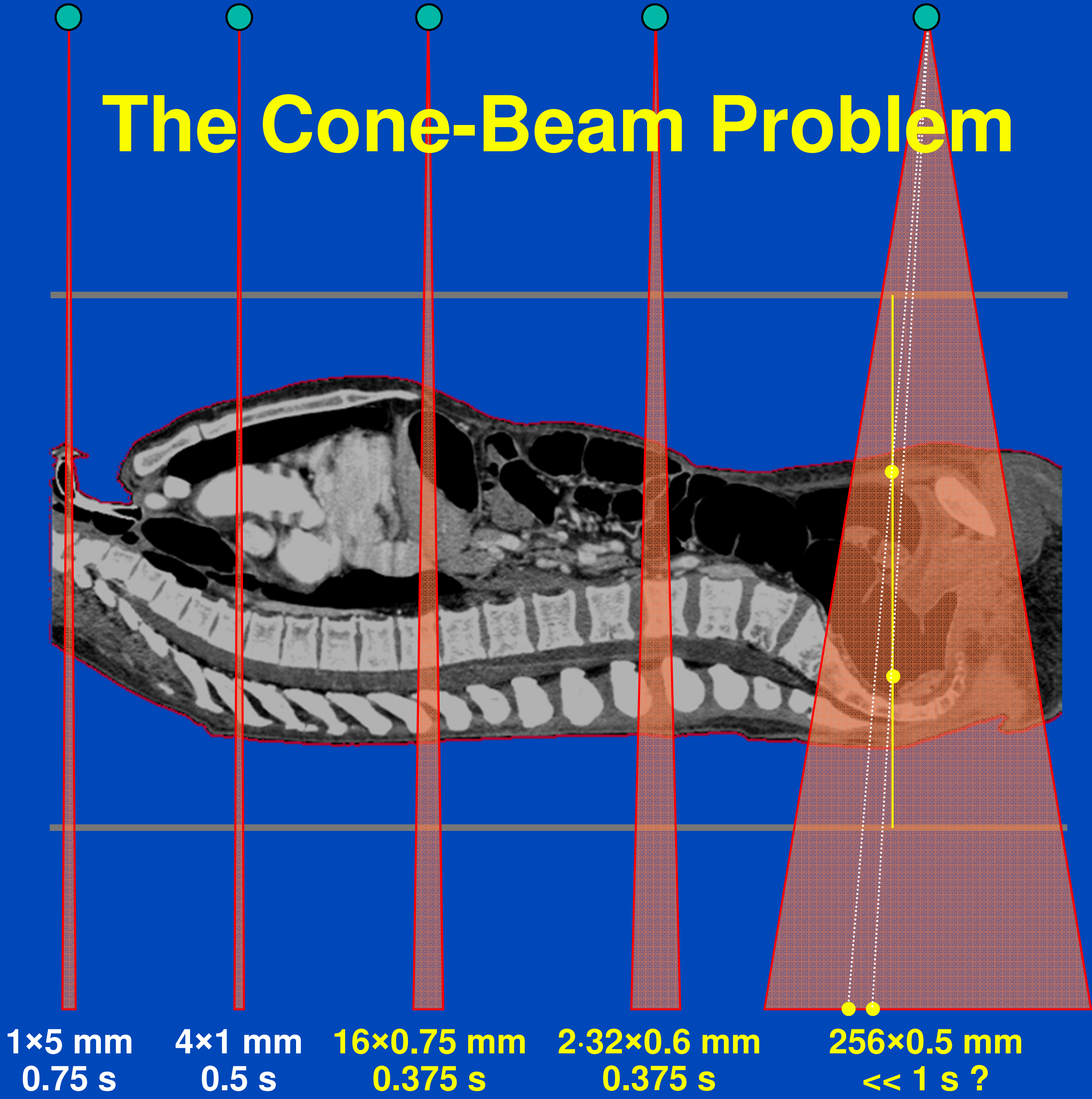
Recommended by and in:

IEC, International Electrotechnical Commision: Medical electrical equipment – 60601 Part 2-44: Particular requirements for the safety of x-ray equipment for computed tomography. Geneva, Switzerland, 1999.

Examples:

- p=1/3=0.333 means that each z-position is covered by 3 rotations (3-fold overlap)
- p=1 means that the acquisition is not overlapping
- $p=p_{max}$ means that each z-position is covered by half a rotation

The Cone-Beam Problem



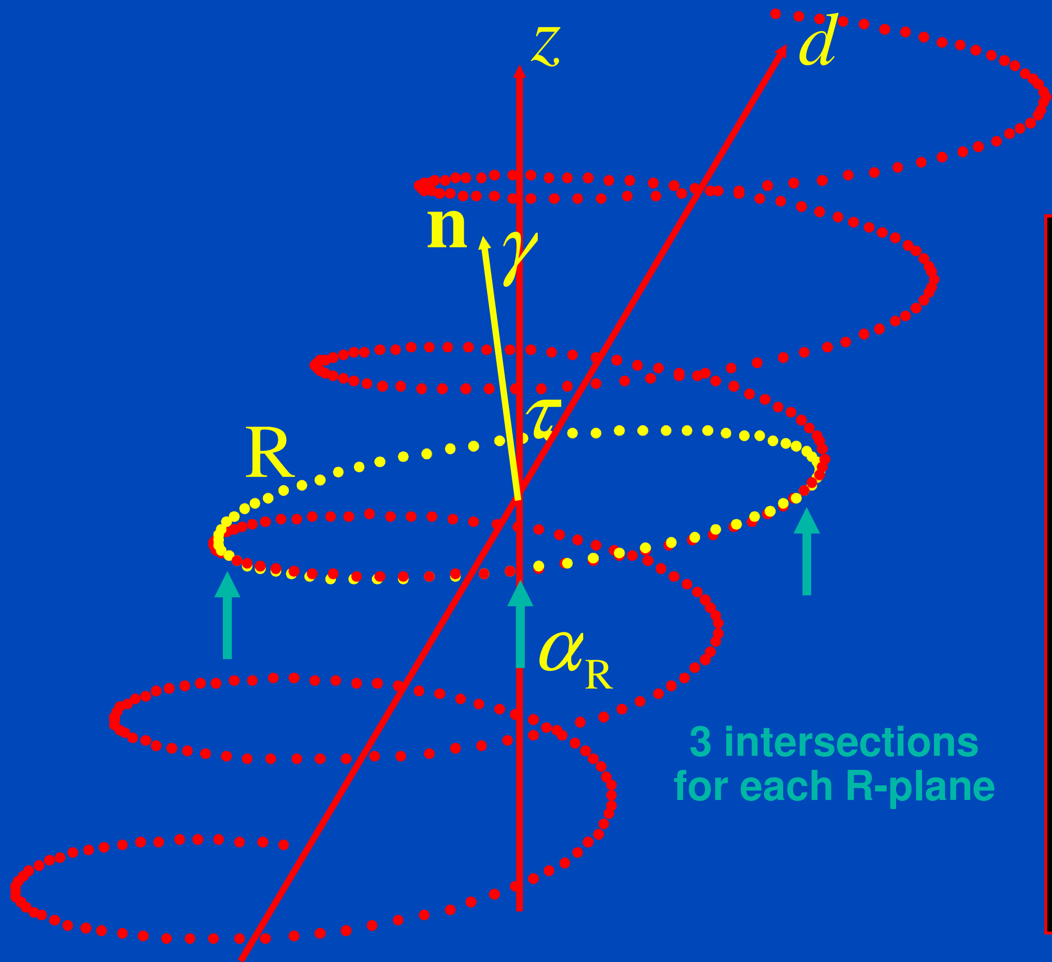
ASSR: Advanced Single-Slice Rebinning

3D and 4D Image Reconstruction for Medium Cone Angles

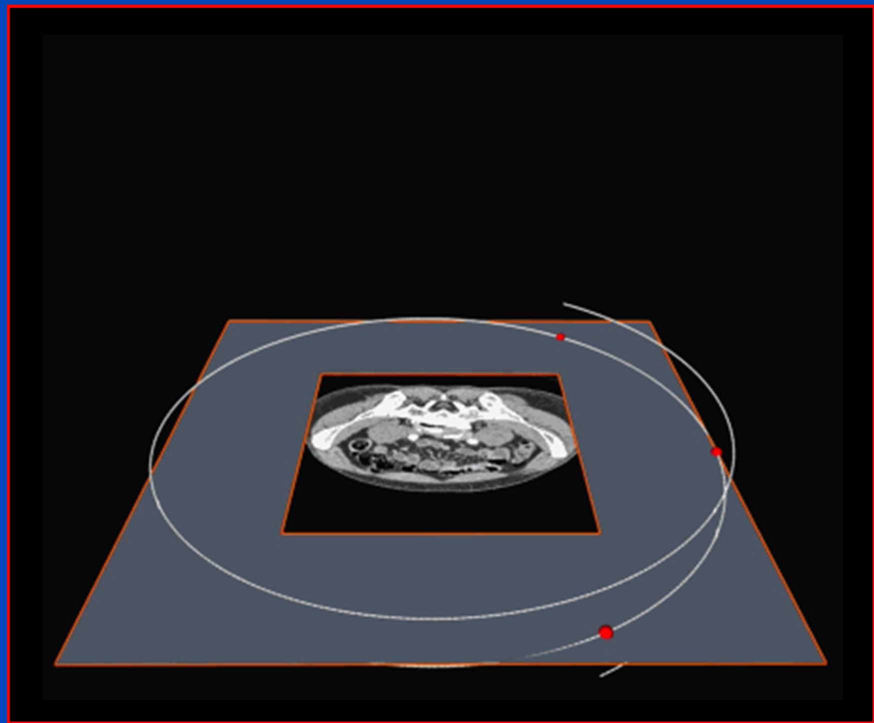
- First practical solution to the cone-beam problem in medical CT
- Reduction of 3D data to 2D slices
- Commercially implemented as AMPR
- ASSR is recommended for up to 64 slices

Do not confuse
the transmission algorithm ASSR
with
the emission algorithm SSRB!

The ASSR Algorithm



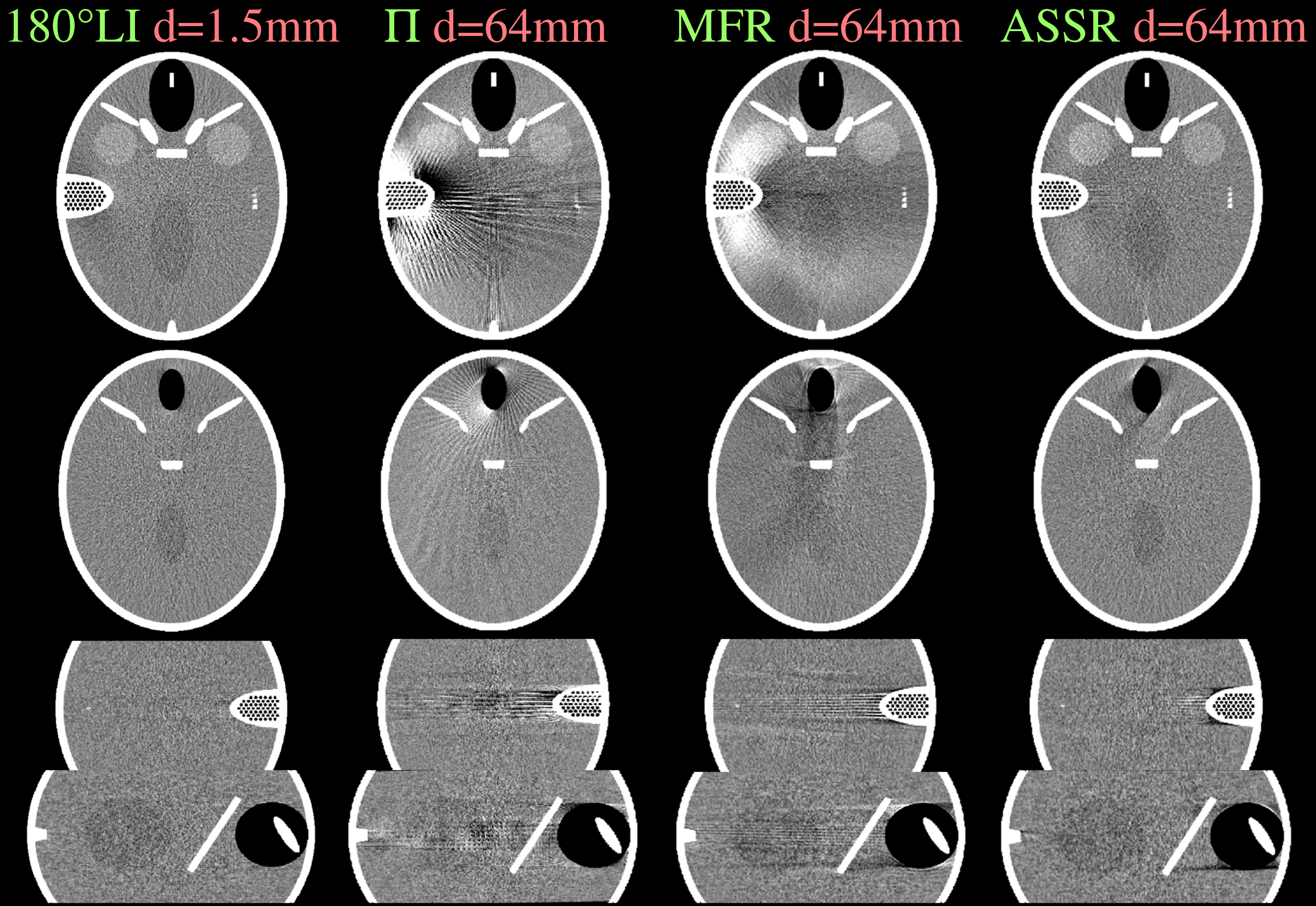
$$p = \frac{d}{MS} \leq 1.5$$



Resulting mean deviation at R_F: $\Delta_{\text{mean}} \approx 0.014d$

at R_M: $\Delta_{\text{mean}} \approx 0.007d$

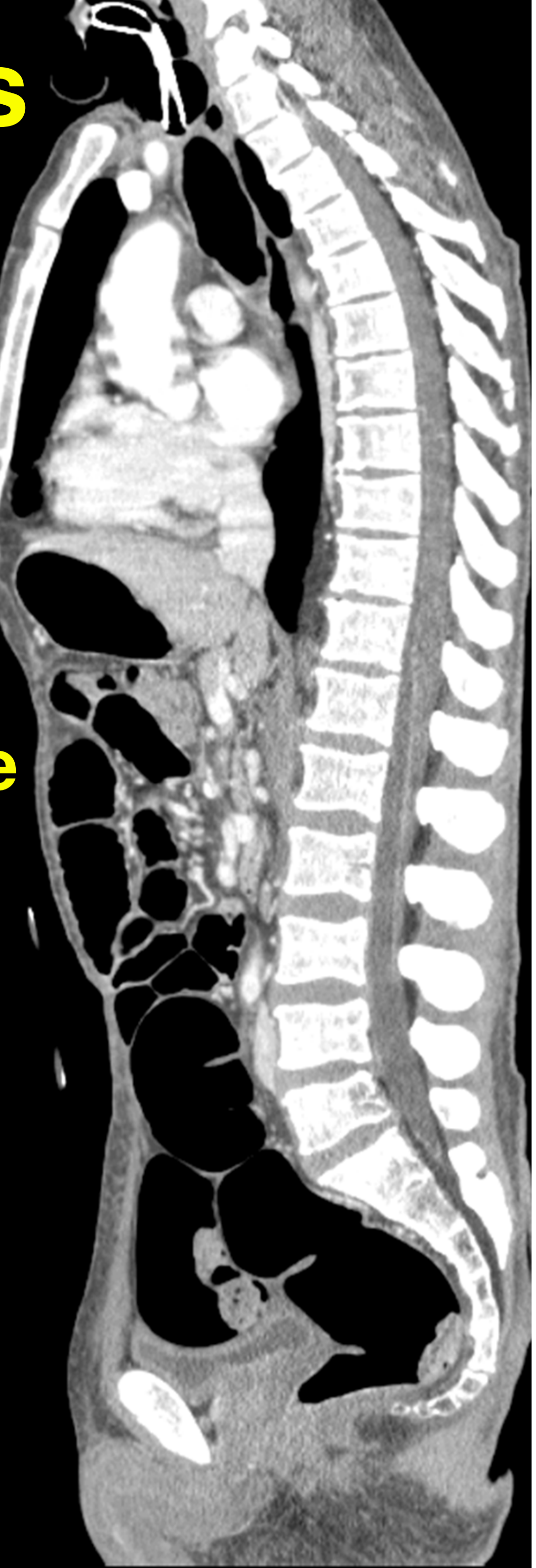
Comparison to Other Approximate Algorithms



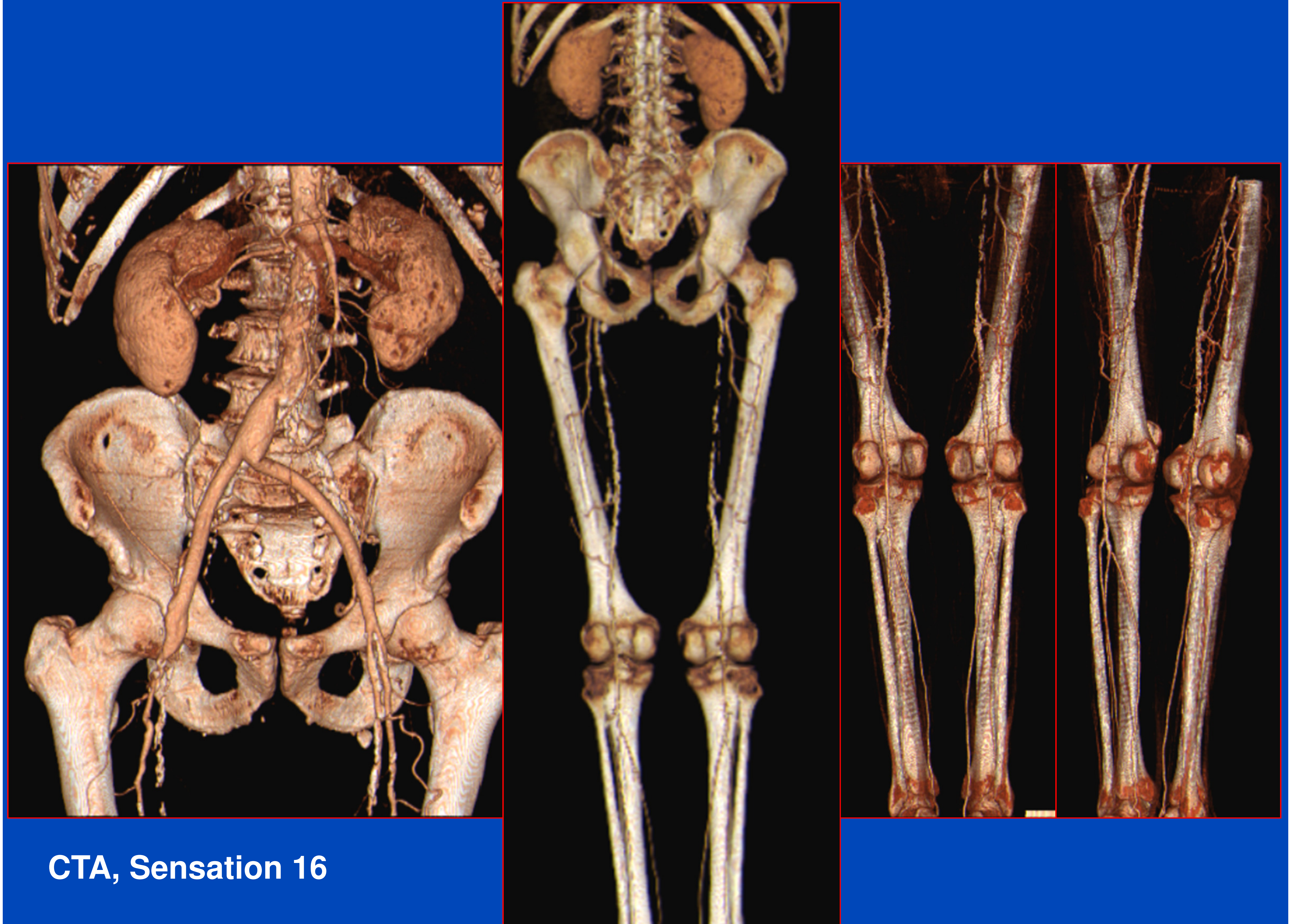
H. Bruder, M. Kachelrieß, S. Schaller. SPIE Med. Imag. Conf. Proc., 3979, 2000

Patient Images with ASSR

- High image quality
- High performance
- Use of available 2D reconstruction hardware
- 100% detector usage
- Arbitrary pitch



- Sensation 16
- 0.5 s rotation
- 16×0.75 mm collimation
- pitch 1.0
- 70 cm in 29 s
- 1.4 GB rawdata
- 1400 images

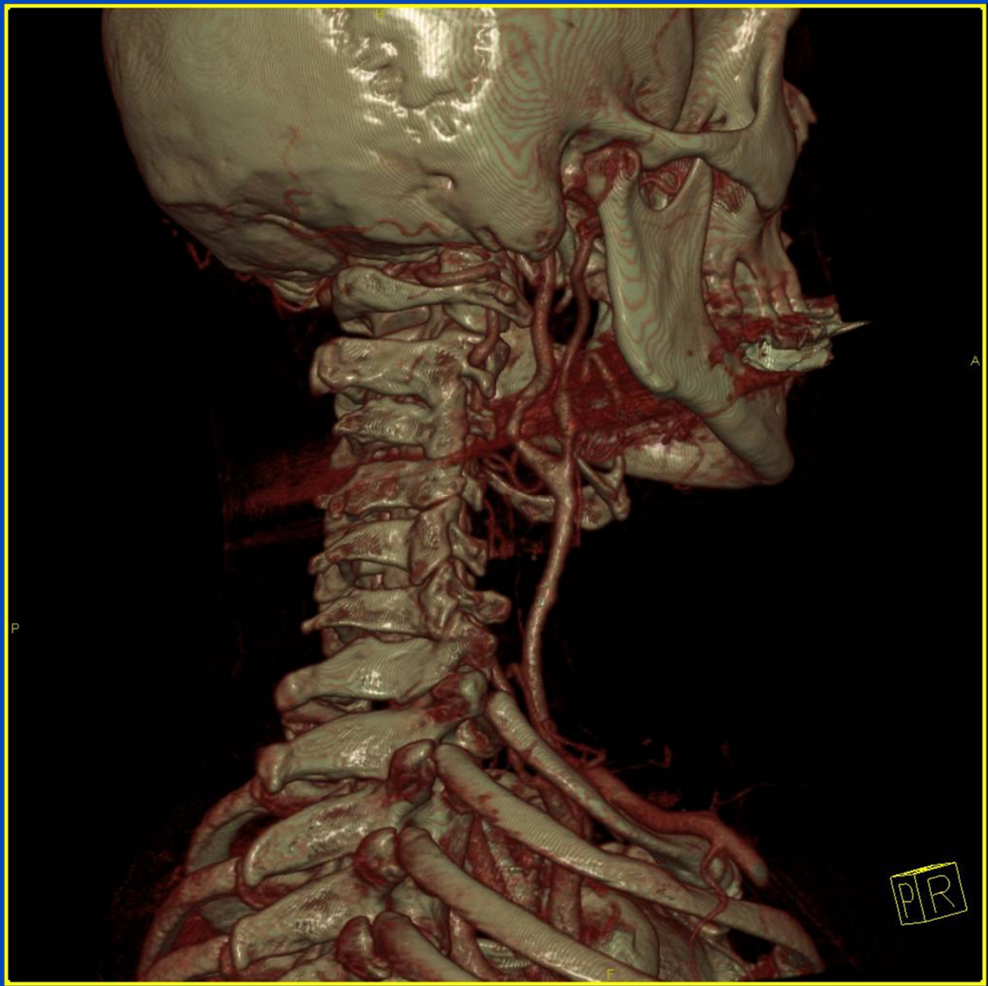


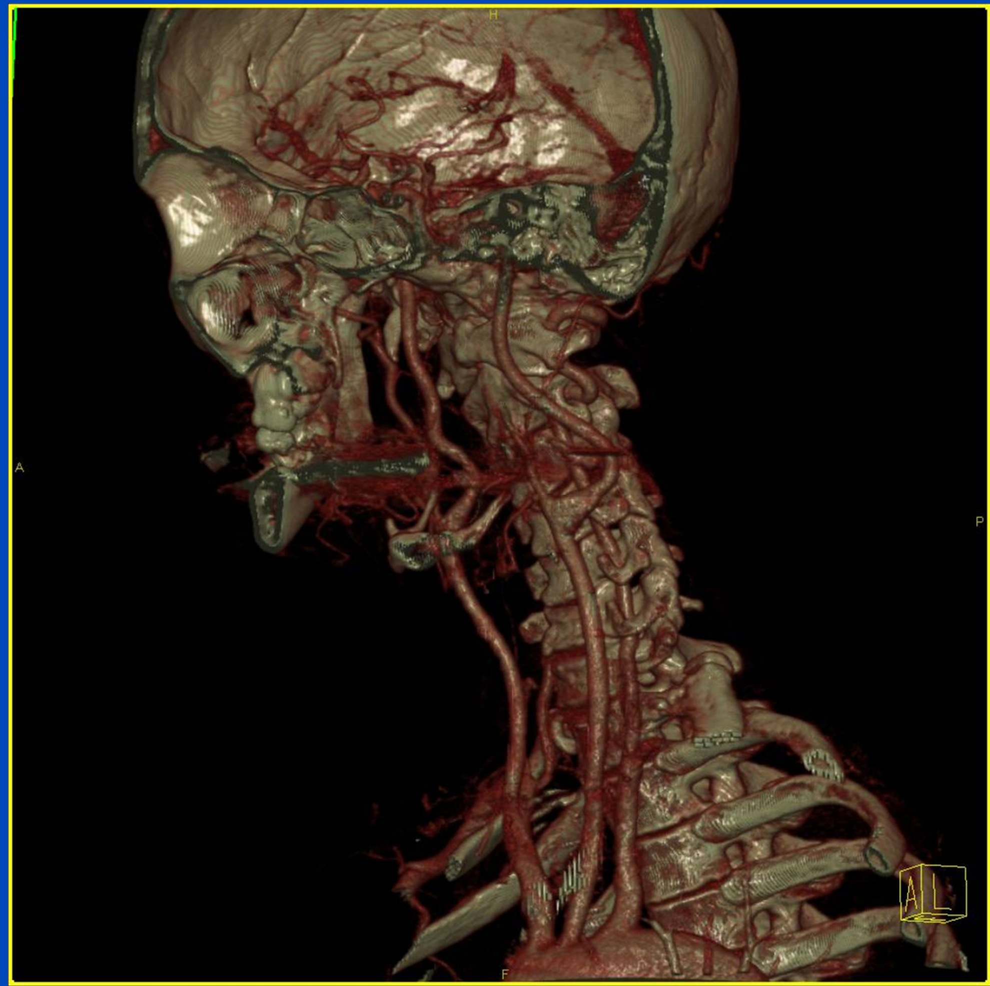
CTA, Sensation 16

Data courtesy of Dr. Michael Lell, Erlangen, Germany

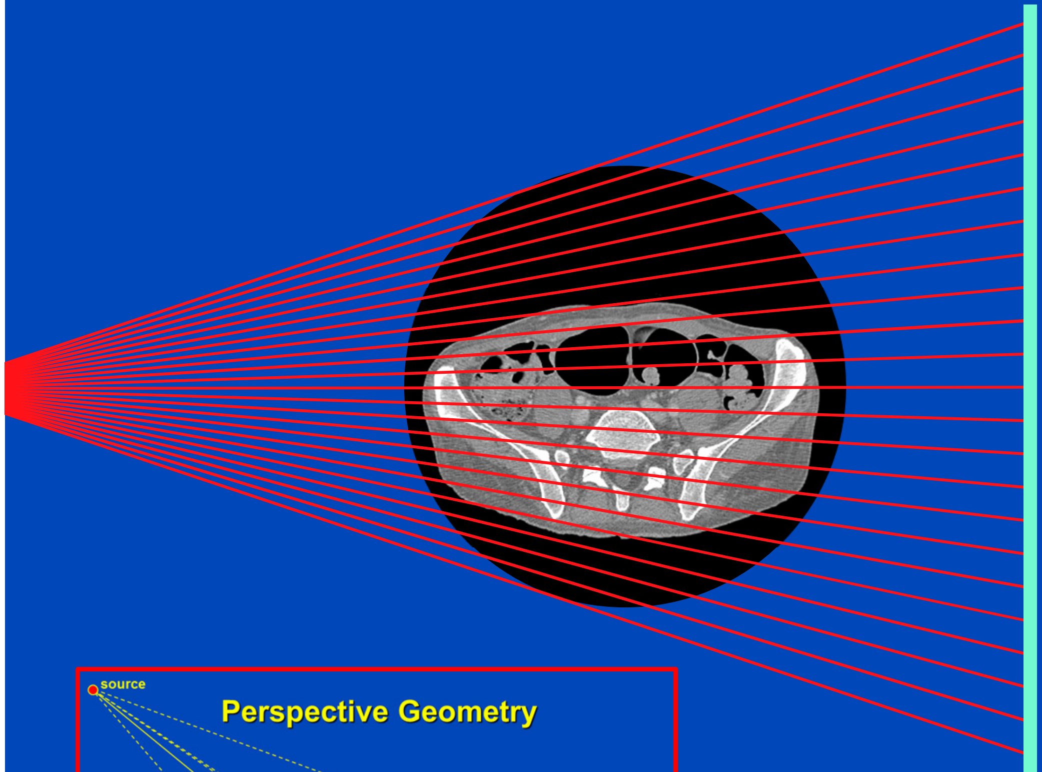
CT-Angiography

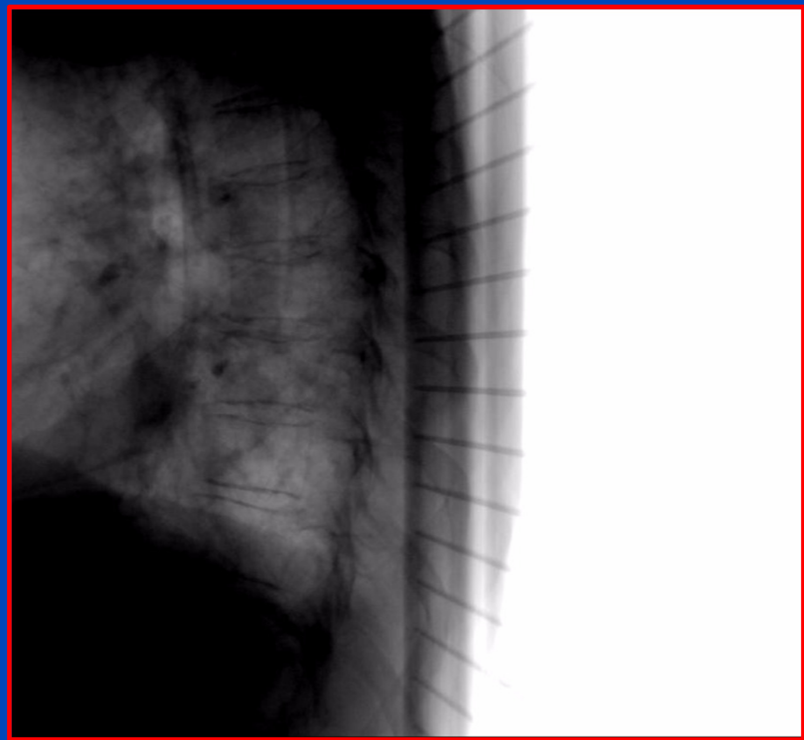
Sensation 64 spiral scan with 2·32×0.6 mm and 0.375 s

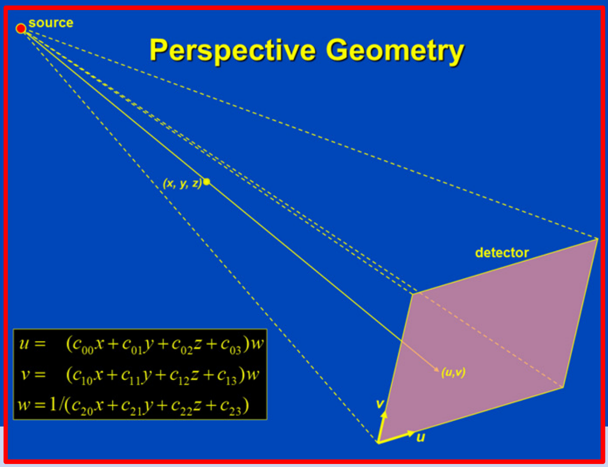




Fully 3D Tomographic Imaging e.g. with Flat Detectors

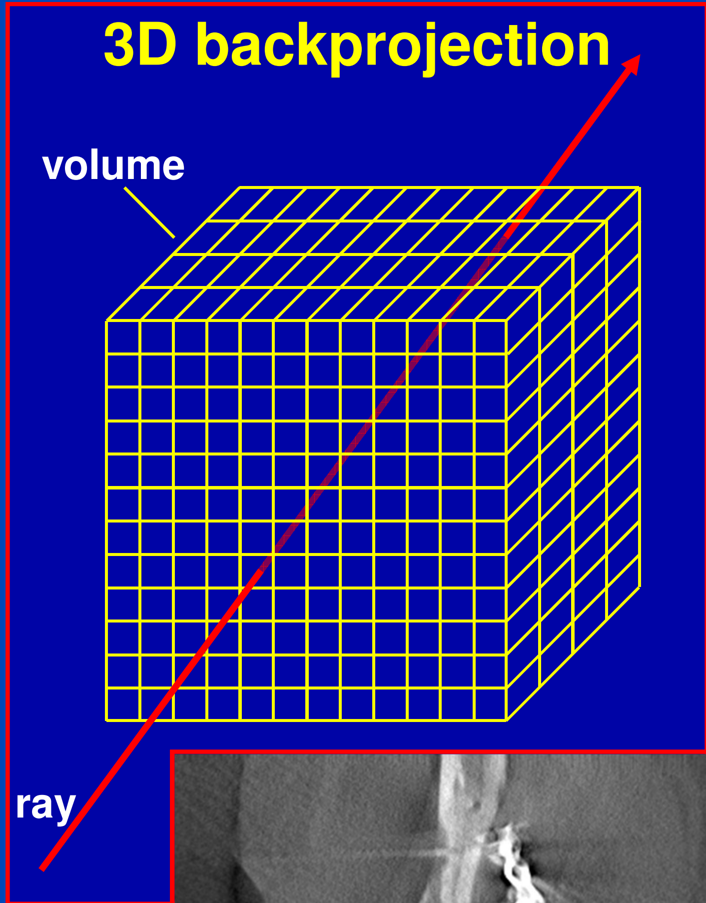


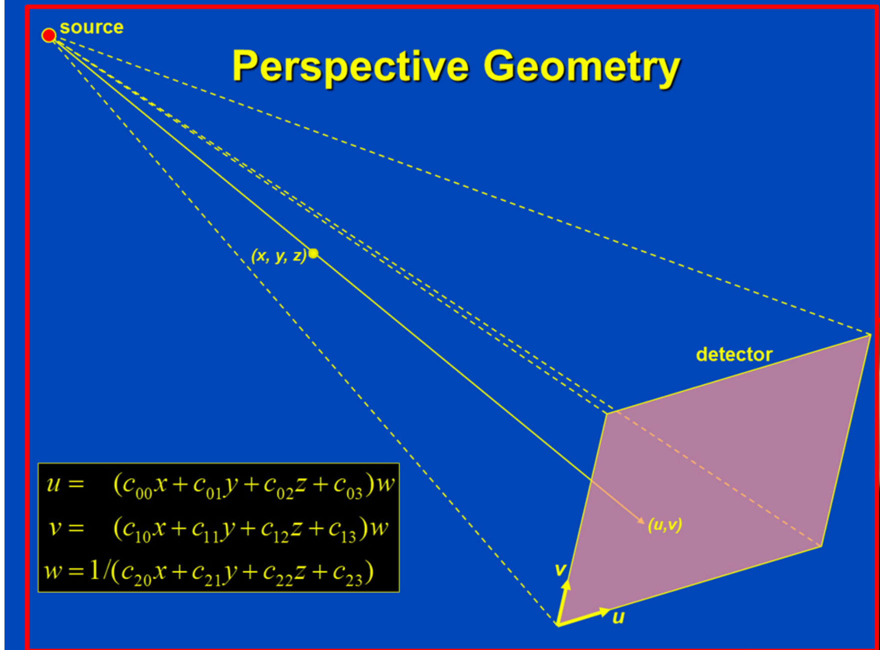




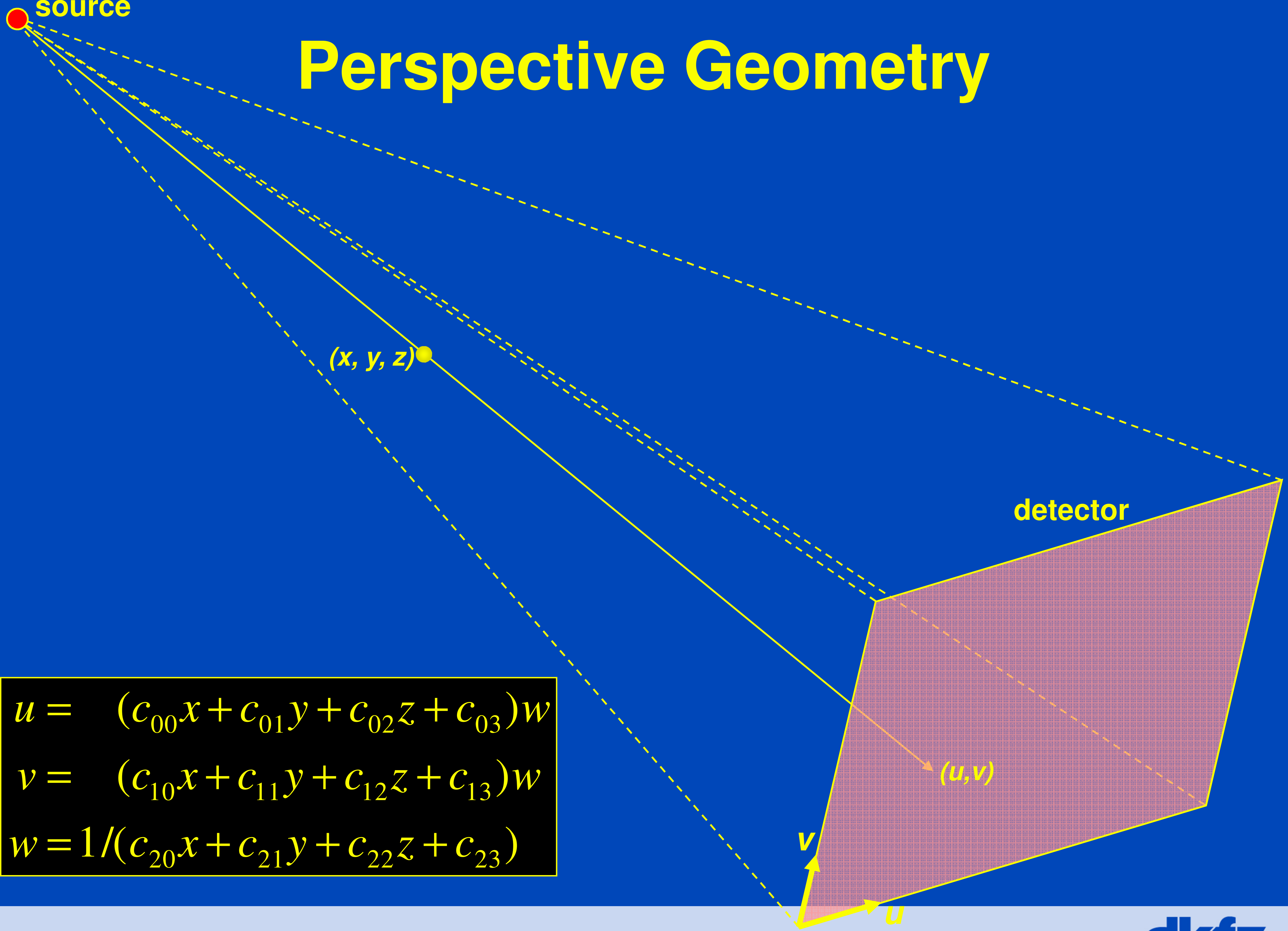
Feldkamp-Type Reconstruction

- **Approximate**
- **Similar to 2D reconstruction:**
  - row-wise filtering of the rawdata
  - followed by backprojection
- **True 3D volumetric backprojection along the original ray direction**





Perspective Geometry



$$u = (c_{00}x + c_{01}y + c_{02}z + c_{03})w$$

$$v = (c_{10}x + c_{11}y + c_{12}z + c_{13})w$$

$$w = 1/(c_{20}x + c_{21}y + c_{22}z + c_{23})$$

Perspective Backprojection: Geometry

voxel position

projection data

$$f(\boldsymbol{r}) = \int d\alpha\, w^2(\alpha,\boldsymbol{r})\, p(\alpha, u(\alpha,\boldsymbol{r}), v(\alpha,\boldsymbol{r}))$$

reconstructed volume

distance weight

$$u(\alpha,\boldsymbol{r}) = \quad (c_{00}x + c_{01}y + c_{02}z + c_{03})w(\alpha,\boldsymbol{r})$$

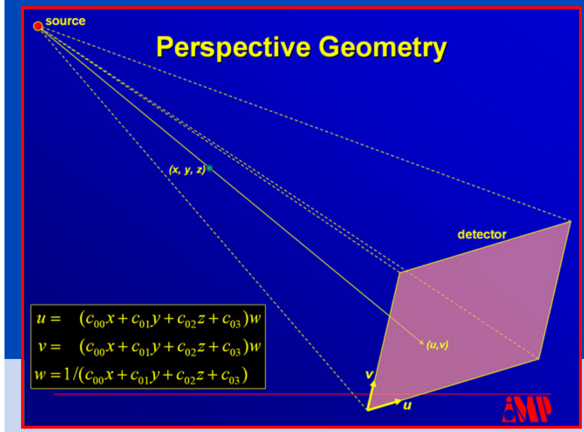
$$v(\alpha,\boldsymbol{r}) = \quad (c_{10}x + c_{11}y + c_{12}z + c_{13})w(\alpha,\boldsymbol{r})$$

$$w(\alpha,\boldsymbol{r}) = 1/(c_{20}x + c_{21}y + c_{22}z + c_{23})$$

$$c_{ij} = c_{ij}(\alpha)$$

trajectory parameter

transform coefficients

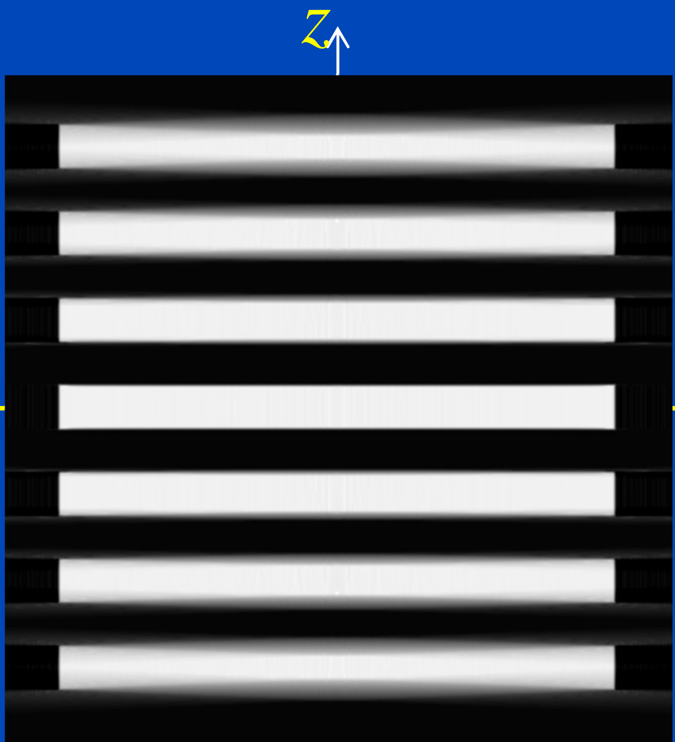


Perspective Backprojection: Reference Implementation

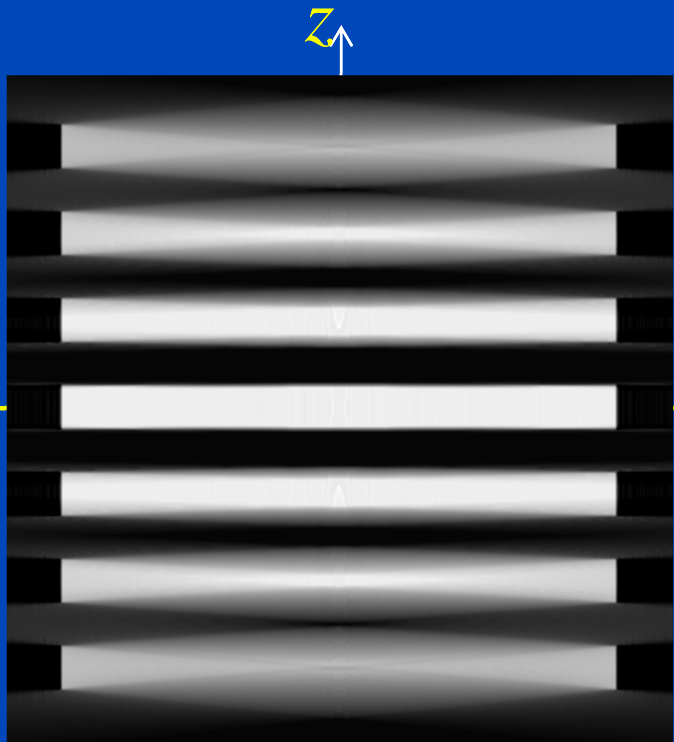
```
void PerBackProjRefLI(float       * const Vol, int const I, int const J, int const K,
                      float const * const Raw, int const N, int const M, int const L,
                      float const * const c00, ..., float const * const c23)
    {
    for(int n=0; n<N; n++) // projection index (alpha)
    for(int i=0; i<I; i++) // slow voxel index (x)
    for(int j=0; j<J; j++) // med. voxel index (y)
    for(int k=0; k<K; k++) // fast voxel index (z)
        {
        float const     w=1/(c20[n]*i+c21[n]*j+c22[n]*k+c23[n]); // distance weight (w)
        float const lreal=w*(c10[n]*i+c11[n]*j+c12[n]*k+c13[n]); // detector row index (v)
        float const mreal=w*(c00[n]*i+c01[n]*j+c02[n]*k+c03[n]); // detector channel index (u)
          int const l =int(lreal); // lower sample position in l
          int const m =int(mreal); // lower sample position in m
        float const wl=lreal-l; // linear interpolation weight in l
        float const wm=mreal-m; // linear interpolation weight in m

        #define V(i, j, k) Vol[((i)*J+j)*K+k] // linear memory layout, use V and
        #define R(n, m, l) Raw[((n)*M+m)*L+l] // R as shortcuts for Vol and Raw

        V(i, j, k)+=w*w*((1-wl)*((1-wm)*R(n, m, l  )+wm*R(n, m+1,   l))   // bilinear interpolation
                        +wl *((1-wm)*R(n, m, l+1)+wm*R(n, m+1, l+1))); // and distance weighting

        #undef V
        #undef R
        }
    }
```

dkfz.

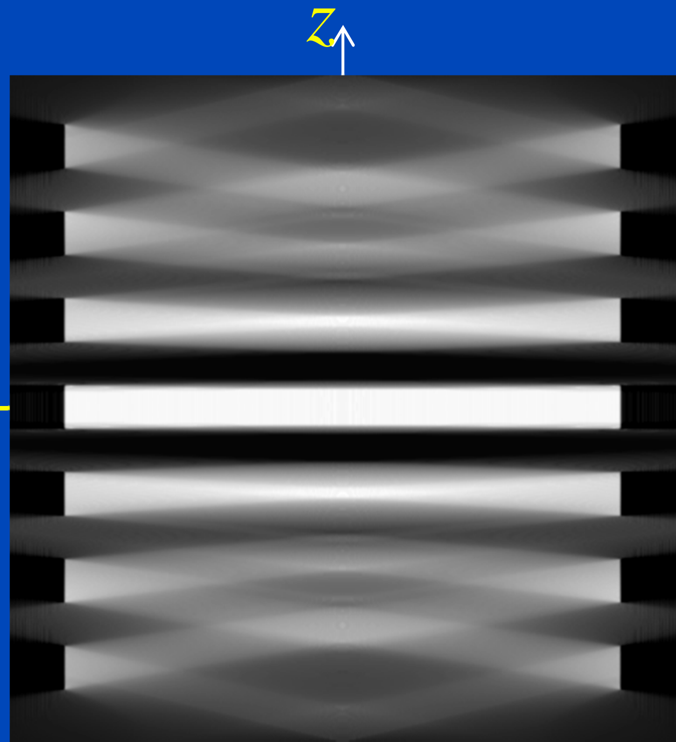
Cone-Beam Artifacts



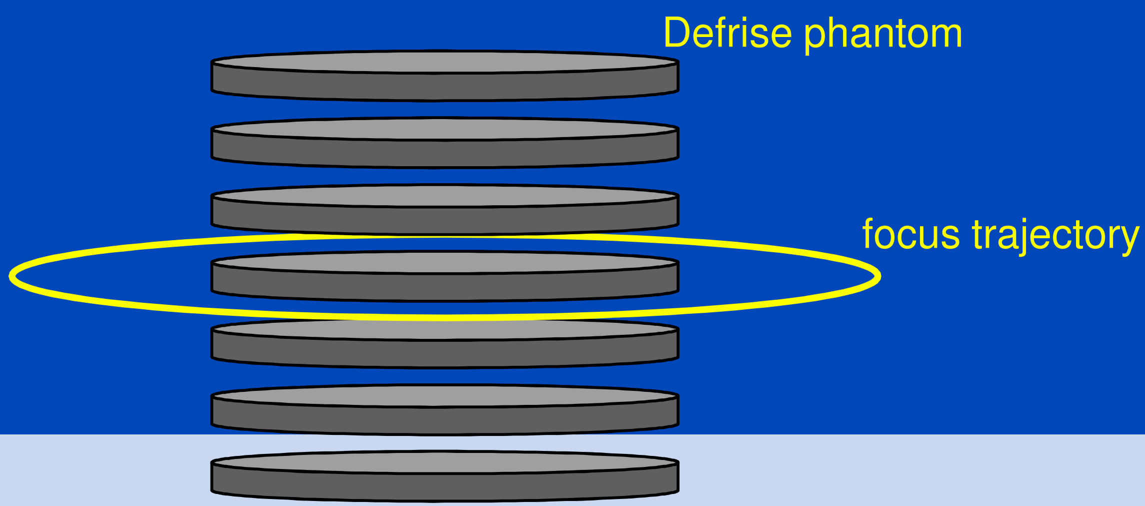
Cone-angle Γ = 6°



Cone-angle Γ = 14°



Cone-angle Γ = 28°

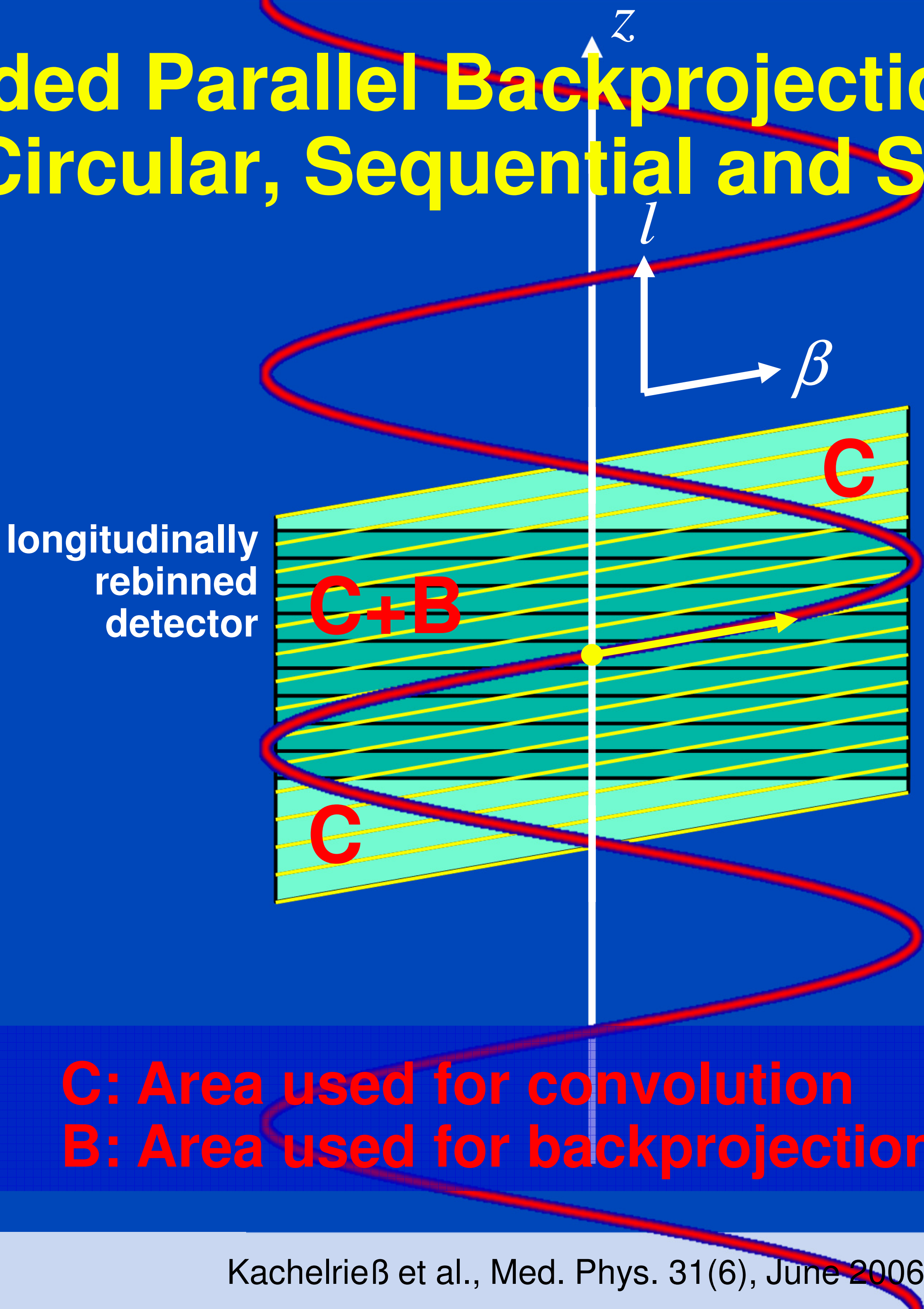


Extended Parallel Backprojection (EPBP)

3D and 4D Feldkamp-Type Image Reconstruction for Large Cone Angles

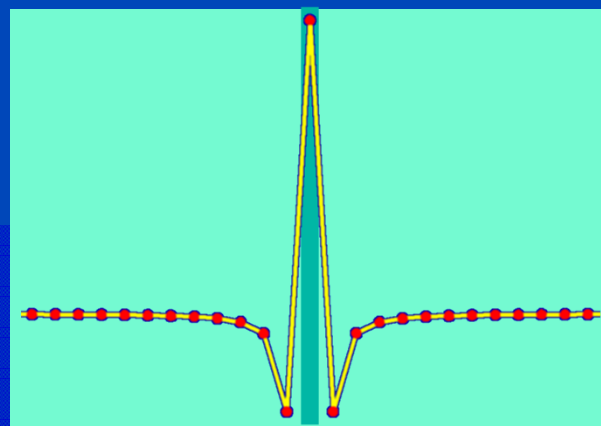
- Trajectories: circle, sequence, spiral
- Scan modes: standard, phase-correlated
- Rebinning: azimuthal + longitudinal + radial
- Feldkamp-type: convolution + true 3D backprojection
- 100% detector usage
- Fast and efficient

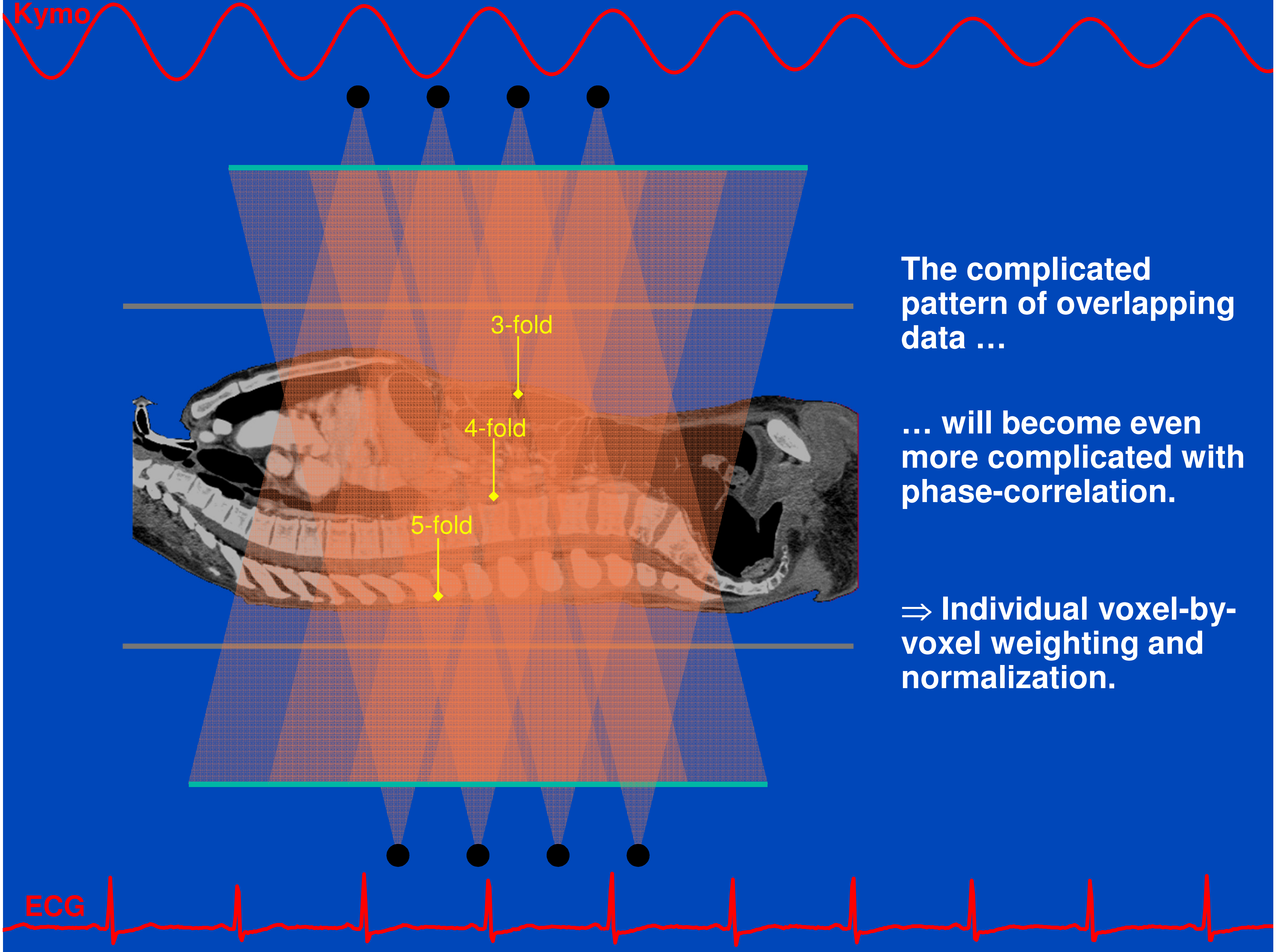
Extended Parallel Backprojection (EPBP) for Circular, Sequential and Spiral CT



$$p = \frac{d}{MS} \leq 1.5$$

C: Area used for convolution
B: Area used for backprojection





The complicated pattern of overlapping data …

… will become even more complicated with phase-correlation.

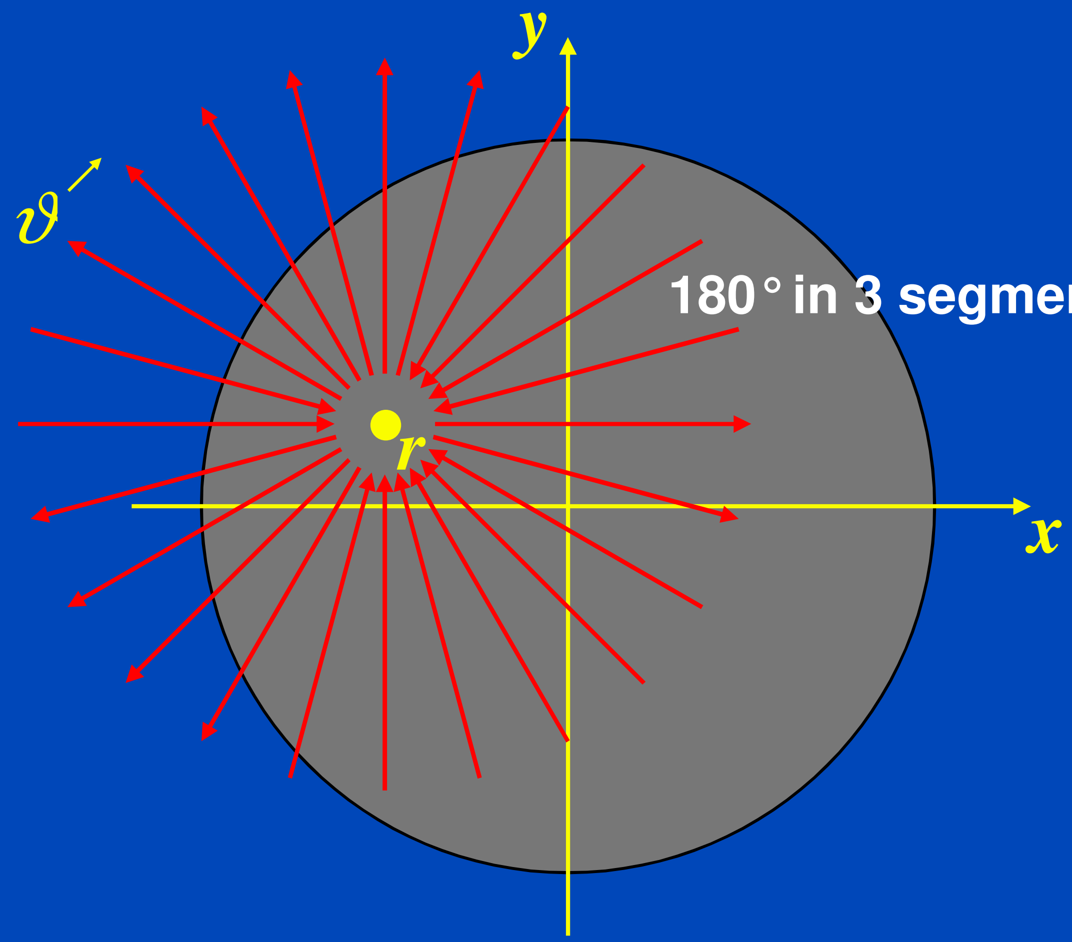
⇒ Individual voxel-by-voxel weighting and normalization.

The 180° Condition

$$\int d\vartheta\, w(\vartheta) = \pi$$

and

$$\sum_k w(\vartheta + k\pi) = 1$$



The (weighted) contributions to each object point must make up an interval of 180° and weight 1.

dkfz.