A General Projection Weight for Feldkamp-Type Cone-Beam Image Reconstruction from Arbitrary CT Scan Trajectories

Michael Knaup, Jan Kuntz, Stefan Sawall, and Marc Kachelrieß
michael.knaup@dkfz.de

Medical Physics in Radiology, German Cancer Research Center (DKFZ), Heidelberg, Germany
www.dkfz.de/ct

Introduction

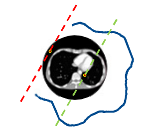
Feldkamp-Davis-Kress (FDK) reconstructions of CT scans [1] require an appropriate pre-weighting of the CT rawdata to account for the redundancies of the measured x-rays. Currently, there exist several weighting schemes for dedicated scan trajectories, e.g. the well-known Parker weighting for circular short scans [2], the shifted-detector weighting [3], the partial scan shifted detector weighting [4], the ROI weighting [5], and the rotate+shift weighting [6]. However, whenever a new scan trajectory is developed, a corresponding weighting scheme must be derived.

In this work we present a general redundancy weighting scheme which can be applied to any arbitrary scan trajectory. Since we are aiming at Feldkamp-type image reconstruction the *z*-components of the trajectory are ignored prior to backprojection and we are rather dealing with an in-plane problem. As long as the trajectory is 180°-complete and as long as it is well sampled, mathematically exact images can be reconstructed in the midplane of the field of measurement (FOM). Off the midplane the well-known Feldkamp-type cone-beam artifacts will occur just as it is the case with a circular scan trajectory. It is understood that the object needs to be laterally fully contained in the FOM. Otherwise truncation artifacts will appear and appropriate detruncation would be required.

Method

Fig. 1 illustrates the derivation of a general weighting scheme in virtual parallel geometry by means of a standard circular short scan. Besides the weights we determine the field of measurement (FOM) of the given scan which might show a complex form for sophisticated scan trajectories. A point (x, y) is inside the FOM, iff for all $\vartheta \in [0, \pi)$ the ray (ϑ, $\xi = x\cos\vartheta + y\sin\vartheta$) is measured at least once, i.e. it has a redundancy $R \geq 1$.

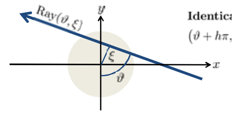
Arbitrary Scan Trajectories



Any straight line through a voxel must be intersected by the source trajectory at least once.

Virtual Parallel Geometry

A ray in virtual parallel geometry is defined by the parameters (ϑ, ξ). It runs through all points (x, y) with $x\cos\vartheta + y\sin\vartheta = \xi$.



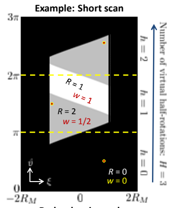
Identical rays:

$(\vartheta + h\pi, (-1)^h\xi) \quad \forall h \in \mathbb{Z}$

Step 1: Redundancies

- For each ray (ϑ, ξ) we count the number of fan projections covering this ray.
- This number of fan projections is called the **redundancy** $R(\vartheta, \xi)$ of this ray.
- The list of fan projections is mapped to a list of **virtual half rotations** h such that adjacent rays in the extended parallel sinogram are covered by adjacent fan projections.

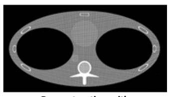
Example: Short scan



Redundancies and corresponding weights in parallel geometry

Step 2: Weights

- Now we look for appropriate weights $0 \leq w(\vartheta, \xi) \leq 1$.
- To account correctly for the redundancies, the weights must fulfill the constraint
$$\sum_{h=0}^{H-1} w\left(\vartheta + h\pi, (-1)^h\xi\right) = 1 \quad \forall \vartheta, \xi$$
- An obvious choice would be
$$w\left(\vartheta + h\pi, (-1)^h\xi\right) = \begin{cases} R(\vartheta,\xi)^{-1} & \text{if ray is measured.} \\ 0 & \text{if ray is not measured.} \end{cases}$$



Reconstruction with non-continuous weights

- However, this non-continuous weights produce unwanted streak artifacts in the final image due to the discreteness of the sampling.
- Therefore, we must smooth the weights under the above constraint.

Reconstruction with smoothed weights

Step 3: Smoothing Weights

We smooth the weights by minimizing the following cost function:

$$C = \sum_{\vartheta,\xi}\left[A_{\vartheta,\xi} + \beta B_{\vartheta,\xi} + \lambda_{\vartheta,\xi}\left(1 - \sum_{h=0}^{H-1} w(\vartheta + h\pi, (-1)^h\xi)\right)\right]$$

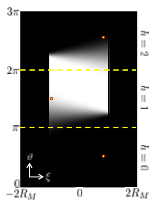
with

$$A_{\vartheta,\xi} = \sum_{h=0}^{H-1}\left[\left(\frac{\partial}{\partial\vartheta} w(\vartheta + h\pi, (-1)^h\xi)\right)^2 + \left(\frac{\partial}{\partial\xi} w(\vartheta + h\pi, (-1)^h\xi)\right)^2\right]$$

$$B_{\vartheta,\xi} = \sum_{h=0}^{H-1}\left[\left(\frac{\partial^2}{\partial\vartheta^2} w(\vartheta + h\pi, (-1)^h\xi)\right)^2 + \left(\frac{\partial^2}{\partial\xi^2} w(\vartheta + h\pi, (-1)^h\xi)\right)^2\right]$$

The cost function will be minimized by a gradient descent approach with respect to the following variables:

- All weights $w(\vartheta, \xi)$ with redundancy $R(\vartheta, \xi) > 1$.
- The Lagrange multipliers $\lambda_{\vartheta,\xi}$ which enforce the constraint.



Smoothed weights

Fig. 1: A general pre-weighting scheme for FDK reconstructions from arbitrary scan trajectories.

Results and Discussion

To demonstrate the ability of our weighting scheme we investigate two sophisticated scan trajectories: On the one hand we consider a rotate+shift scan composed of a 165° elliptical trajectory preceeded and succeeded by a shift. This scan trajectory is realized in the Ziehm Vision RFD 3D mobile C-arm system (Ziehm Imaging GmbH, Nürnberg, Germany) and allows for a 180°-complete data acquisition with a physical rotation range of only 165° [6]. On the other hand we investigate a full scan with virtual isocenter where source and detector rotate independently on two isocentric circles, as they are used in the patient alignment imaging ring (PAIR) system (medPhoton GmbH, Salzburg, Austria).

Fig. 2 shows the resulting FOMs and general weights for these trajectories as well as image reconstructions from both simulated and measured rawdata. The rotate+shift scan has almost no redundancies, therefore the weights are mostly 1. Only in some small areas there is a freedom to choose weights smaller than 1 and for smoothing. In contrast, the redundancy of the PAIR scan is mostly 2. Here, there is more freedom of choice which results in a more complex weight sinogram. The reconstructed images show neither low frequency artifacts due to incorrect consideration of the redundancies nor streak artifacts due to unsteady weights. This demonstrates that our general weighting scheme works correctly for all trajectories examined.

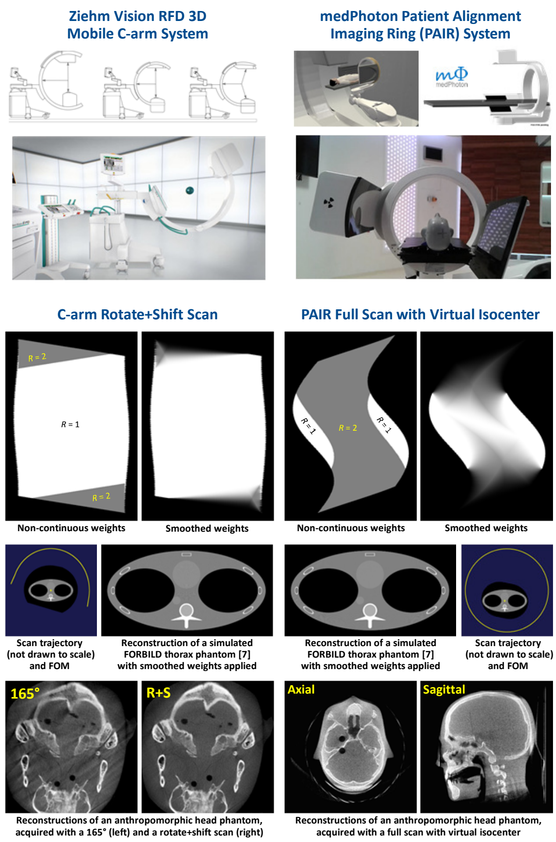


Fig. 2: Results. Left: Rotate+shift scan of a Vision RFD 3D mobile C-arm system (Ziehm Imaging GmbH, Nürnberg, Germany). Right: Full scan with virtual isocenter of a patient alignment imaging ring (PAIR) system (medPhoton GmbH, Salzburg, Austria).

Summary and Conclusion

We developed a new redundancy weighting scheme for Feldkamp-type CT image reconstruction. While existing weighting schemes only work for dedicated scan trajectories, the proposed scheme allows reconstructions for arbitrary source and detector trajectories. This significantly eases the development of new sophisticated trajectories, since the derivation of a corresponding dedicated weighting scheme is no longer required.

In this work we applied the general weighting scheme to a standard short scan, a rotate+shift scan of a Ziehm Vision RFD 3D mobile C-arm system, and a full scan with virtual isocenter of a patient alignment imaging ring (PAIR) system. We demonstrated that the weighting works correctly for both the standard case as well as the sophisticated cases.

Acknowledgments

This work was supported in parts by the Deutsche Forschungsgemeinschaft (DFG) under grant KA 1678/11-1. The high performance image reconstruction software was provided by RayConStruct® GmbH, Nürnberg, Germany.

[1] L. Feldkamp, L. Davis, and J. Kress, "Practical cone-beam algorithm," Journal of the Optical Society of America, vol. 1, no. 6, pp. 612-619 (1984).
[2] D. L. Parker, "Optimal short scan convolution reconstruction for fan-beam CT," Med. Phys., vol. 9, no. 2, pp. 254-257 (1982).
[3] G. Wang, "X-ray micro-CT with a displaced detector array," Med. Phys., vol. 29, no. 7, pp. 1634-1636 (2002).
[4] C. Maaß, M. Knaup, R. Lapp, M. Karolczak, W. A. Kalender, and M. Kachelrieß, "A new weighting function to achieve high temporal resolution in circular cone-beam CT with shifted detectors," Med. Phys., vol. 35, no. 12, pp. 5898-5909 (2008).
[5] C. Maaß, M. Knaup, and M. Kachelrieß, "New approaches to region of interest computed tomography," Med. Phys., vol. 38, no. 6, pp. 2868-2878 (2011).
[6] L. Ritschl, J. Kuntz, and M. Kachelrieß, "The rotate-plus-shift C-arm trajectory part I+II," submitted to Med. Phys. (2015).
[7] M. Kachelrieß, "The FORBILD CT–simulation phantoms," Proc. of the 1999 Int. Meeting on Fully 3D Image Reconstruction, p. 383 (1999).